Dark Energy from Inhomogeneities in f(R) or Scalar-Tensor Gravity

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Azores '14
Friday 6th June
2014

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Introduction

After large-scale averaging, universe is *homogeneous and isotropic,* but averaging inhomogeneities changes physics, giving effective fluids.

$$G^{(0)}_{\mu\nu} + \Lambda g^{(0)}_{\mu\nu} = \kappa T^{(0)}_{\mu\nu} + \kappa t^{(0)}_{\mu\nu}$$

Small non-zero vacuum energy difficult to justify from quantum field theory.

Using Green & Wald (2011) scheme, back-reaction in Einstein gravity is radiation-like.

Starobinsky R^2 term motivated by quantum gravity and inflation.

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2} - 2\Lambda\right) + \mathcal{L}_{Matter}\right]$$

f(R) equivalent to scalar-tensor theory, which is easier to work with.

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa}\left(\phi R - V(\phi)\right) + \mathcal{L}_{matter}\right]$$

Inhomogeneity Scheme

$$h_{\mu\nu}(\lambda, x) := g_{\mu\nu}(\lambda, x) - g^{(0)}_{\mu\nu}(x)$$

$$\phi(\lambda) := \phi_0 + \phi_p(\lambda)$$

$$R^{(1)}_{\mu\nu\alpha\beta} := 2\nabla_{[\mu|}\nabla_{[\nu}h_{\alpha]\,|\beta]}$$

Metric and scalar field split into background and perturbation pieces, with a single inhomogeneity parameter, λ.

Perturbation length scale << averaging scale, so λ→0 in “weak limit”.

$$h \sim \lambda \sin(x/\lambda)$$

$$\nabla_{\mu_1}\ldots\nabla_{\mu_m}h_{\rho_1\sigma_1}\ldots h_{\rho_n\sigma_n} \sim \lambda^{n-m}$$

Scale M with λ^{-1}, effectively setting the inhomogeneity length to this scale.

$$\frac{\nabla_\mu\nabla_\nu}{M^2}h_{\rho\sigma} \sim h_{\rho\sigma}$$

$$\nabla_{\mu_1}\ldots\nabla_{\mu_n}\phi_p \sim \nabla_{\mu_1}\ldots\nabla_{\mu_n}h_{\rho\sigma}$$

Effective Stress-energy tensors

Stress-energy tensor can be split into traceless & pure trace parts.

$$t^{(0)}_{\mu\nu} = \underbrace{t^{(0)}_{\mu\nu} - \frac{1}{4} g^{(0)}_{\mu\nu} t^{(0)}}_{\text{traceless}} + \underbrace{\frac{1}{4} g^{(0)}_{\mu\nu} t^{(0)}}_{\text{pure trace}}$$

$$\kappa t^{E}_{\mu\nu} \underset{weak}{=} -\frac{1}{2} h^{\alpha\beta} R^{(1)}_{\mu\nu\alpha\beta}$$

Einstein case is traceless, weak-energy satisfying, gauge invariant, radiation only.

$$\kappa t^{(0)} \underset{weak}{=} -\frac{R^{(1)2}}{6M^2}$$

Starobinsky case has a negative pure trace part, correct sign for dark energy.

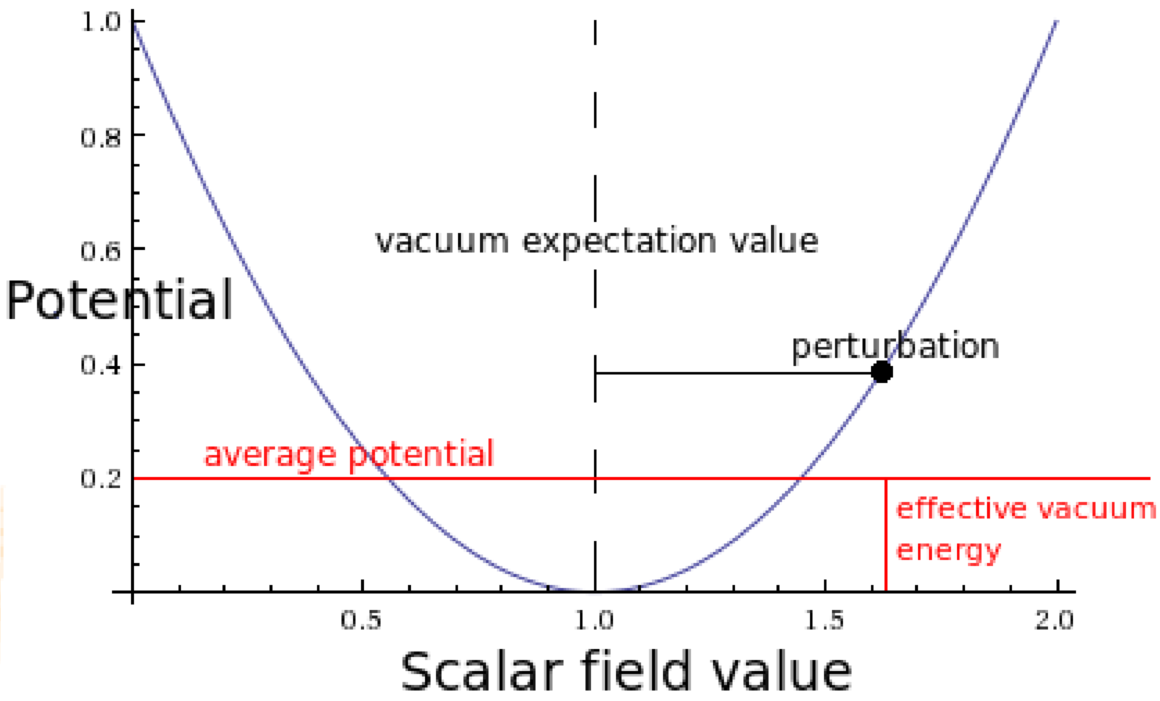
$$\phi_0 \mathcal{G}^{(0)}_{\mu\nu} \underset{weak}{=} \kappa T^{(0)}_{\mu\nu} + \underbrace{\phi_0 \kappa t^{E(0)}_{\mu\nu}}_{traceless} - \underbrace{\frac{1}{2} g^{(0)}_{\mu\nu} V^{(0)}}_{\text{pure trace}}$$

Scalar-tensor description is more intuitive.

Interpretation

Jordan-frame scalar-tensor potential relating to Starobinsky gravity:

$$V(\phi) = \frac{3M^2}{2}(\phi - 1)^2$$



Inhomogeneity at high-frequency creates oscillations, raising the average potential from its minimum.

Oscillation is coupled to matter and gravitational waves, so forcing and damping effects can occur.

Largest effect from perturbation length approaching the length scale of the theory, which, in inflation case, is close to GUT scale.

Effective scale factor dependence depends on the physics behind the inhomogeneity. Might address the “why now?” problem.

Summary

It is difficult to justify a small positive cosmological constant from quantum field theory.

If we suppose the contribution from vacuum energy is zero, we have to account for dark energy some other way.

Backreaction from inhomogeneities can imitate effective fluids.

In Einstein gravity, the effective fluid is radiation-like.

With an extra scalar mode, a pure trace component with the correct sign and order to explain dark energy appears.

The most significant perturbation length depends on the parameters of the scalar potential and the dependence on scale factor depends on the source of inhomogeneity.