

School Observational Cosmology Angra – Terceira – Açores 5th June 2014 Juan García-Bellido Física Teórica UAM Madrid, Spain

Outline

Lecture 2

- Density and GW power spectra
- Inflationary Model Building
- CMB & LSS: predictions from inflation
- Quantum to classical transition
- Reheating after inflation

Ripples in Space





SCALAR POWER SPECTRA

two-point correlation function in Fourier space

$$\langle 0 | \mathcal{R}_k^* \mathcal{R}_{k'} | 0 \rangle = \frac{|u_k|^2}{z^2} \, \delta^3 (\mathbf{k} - \mathbf{k}') \equiv \frac{\mathcal{P}_{\mathcal{R}}(k)}{4\pi k^3} \, (2\pi)^3 \, \delta^3 (\mathbf{k} - \mathbf{k}') \,,$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|u_k|^2}{z^2} = \frac{\kappa^2}{2\epsilon} \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{3-2\nu} \equiv A_S^2 \left(\frac{k}{aH}\right)^{n_s-1}$$

$$\mathcal{R}_k = \zeta_k = \frac{u_k}{z} \quad \text{enter the horizon at } a = k/H$$

$$\mathcal{A}_S^2 = \frac{\kappa^2}{2\epsilon} \left(\frac{H}{2\pi}\right)^2 = \frac{1}{\pi\epsilon} \frac{H^2}{M_P^2} \quad \text{amplitude and tilt,}$$

$$scale invariant$$

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{d \ln k} = 3 - 2\nu = 2 \left(\frac{\delta - 2\epsilon}{1 - \epsilon}\right) \simeq 2\eta_V - 6\epsilon_V \ll 1$$

$$running of the tilt$$

$$\frac{dn_s}{d\ln k} = -\eta \mathcal{H} \left(2\xi + 8\epsilon^2 - 10\epsilon\delta \right) \simeq 2\xi_V + 24\epsilon_V^2 - 16\eta_V\epsilon_V$$

TENSOR POWER SPECTRA
tensor (gravitational wave) metric perturbation

$$\sum_{\lambda} \langle 0 | h_{k,\lambda}^* h_{k',\lambda} | 0 \rangle = 4 \frac{2\kappa^2}{a^2} |v_k|^2 \delta^3(\mathbf{k} - \mathbf{k}') \equiv \frac{\mathcal{P}_g(k)}{4\pi k^3} (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}')$$

$$\mathcal{P}_g(k) = 8\kappa^2 \left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{3-2\mu} \equiv A_T^2 \left(\frac{k}{aH}\right)^{n_T}$$

$$h_k = \kappa \sqrt{2} \frac{v_k}{a} \quad \text{enter the horizon at } a = k/H$$

$$A_T^2 = 8\kappa^2 \left(\frac{H}{2\pi}\right)^2 = \frac{16}{\pi} \frac{H^2}{M_P^2} \quad \text{amplitude and tilt,}$$

$$\text{scale invariant}$$

$$n_T \equiv \frac{d \ln \mathcal{P}_g(k)}{d \ln k} = 3 - 2\mu = \frac{-2\epsilon}{1 - \epsilon} \simeq -2\epsilon_V < 0 \quad \ll 1$$
running of the tilt
$$\frac{dn_T}{d \ln k} = -\eta \mathcal{H} \left(4\epsilon^2 - 4\epsilon\delta\right) \simeq 8\epsilon_V^2 - 4\eta_V \epsilon_V$$





nfation **MOCE** Building

Single field inflation



single field inflation



INFLATIONARY MODEL BUILDING

slow-roll parameters

$$\epsilon = \frac{1}{2\kappa^2} \left(\frac{V'}{V}\right)^2 \,,$$

$$\xi = \frac{1}{\kappa^4} \left(\frac{V'V'''}{V^2} \right) \,,$$

$$\eta = \frac{1}{\kappa^2} \left(\frac{V''}{V} \right) ,$$
$$N = \int_{\phi_{\text{end}}}^{\phi} \frac{\kappa d\phi}{\sqrt{2\epsilon}} ,$$

amplitude and tilt

$$A_S = \frac{\kappa}{\sqrt{2\epsilon}} \frac{H}{2\pi}, \qquad n_s = 1 + 2\eta - 6\epsilon,$$

$$A_T = 2\sqrt{2\kappa} \frac{H}{2\pi}, \qquad n_T = -2\epsilon,$$

running of the tilt

$$\frac{d\,n_s}{d\,\ln k} = 2\xi + 24\epsilon^2 - 16\epsilon\eta\,,$$

consistency relation $r = 16\epsilon = -8 n_T$.

SIMPLE EXACT MODEL

POWER-LAW INFLATION $V(\phi) = V_0 e^{-\beta \kappa \phi}$ $\beta \ll 1$ Hamilton-Jacobi equation $3H^2(\phi) = \frac{2}{\kappa^2} \left(\frac{\partial H}{\partial \phi}\right)^2 + \kappa^2 V(\phi) \qquad \beta H = \kappa \dot{\phi}$ Exact solution $H(\phi) = H_0 e^{-\frac{1}{2}\beta\kappa\phi}$ $H_0^2 = \frac{\kappa^2}{3}V_0\left(1 - \frac{\beta^2}{6}\right)^{-1}$ $V_0 \equiv M^4$ $\epsilon = \frac{2}{\kappa^2} \left(\frac{H'}{H}\right)^2 = \frac{1}{2}\beta^2 < 1$ $\delta = \frac{2}{\kappa^2} \left(\frac{H''}{H}\right)^2 = \frac{1}{2}\beta^2 < 1$ $a \propto t^p \xrightarrow{\epsilon=1/p} p = \frac{2}{\beta^2}$ $\epsilon = \delta = \frac{1}{p} = \text{const}$ $\xi = \frac{1}{\kappa^4} \left(\frac{H' H'''}{H^2} \right) = \frac{1}{4} \beta^4 < 1 \qquad N = \int_{-\infty}^{\phi_{\text{end}}} \frac{\kappa d\phi}{\sqrt{2\epsilon}} = \frac{\kappa}{\beta} (\phi_{\text{end}} - \phi) = 65$ $A_S = \frac{\kappa}{\sqrt{2\epsilon}} \frac{H}{2\pi} = 5 \times 10^{-5} \implies M \simeq 10^{-3} M_P \simeq M_{\rm GUT}$ $n_s - 1 = 2\left(\frac{\delta - 2\epsilon}{1 - \epsilon}\right) = -\frac{2}{p - 1} \qquad n_s = 0.96 \implies p = 51 \qquad \frac{d n_s}{d \ln k} = 0$ $n_T = -\frac{2\epsilon}{1-\epsilon} = -\frac{2}{p-1} = n_s - 1 \qquad r = -8 n_T = 0.32$ ruled out !

CHAOTIC INFLATION MODEL

$$V(\phi) = \frac{1}{2}m^2\phi^2 \qquad \qquad H^2 \simeq \frac{\kappa^2}{6}m^2\phi^2$$

$$\begin{aligned} \epsilon &= \frac{1}{2\kappa^2} \left(\frac{V'}{V}\right)^2 = \frac{2}{\kappa^2 \phi^2} = 1 & \Rightarrow & \phi_{\text{end}} = \frac{M_P}{2\sqrt{\pi}} \simeq \frac{M_P}{3.5} \\ \eta &= \frac{1}{\kappa^2} \left(\frac{V''}{V}\right) = \frac{2}{\kappa^2 \phi^2} = \epsilon = \frac{1}{2N} & \xi = 0 \end{aligned}$$

$$N &= \int_{\phi_{\text{end}}}^{\phi} \frac{\kappa d\phi}{\sqrt{2\epsilon}} = \left(\frac{\kappa\phi}{2}\right)^2 \Big|_{\phi_{\text{end}}}^{\phi} \approx \frac{\kappa^2 \phi^2}{4} & \Rightarrow & \phi_{60} = 3M_P \\ A_S &= \frac{\kappa m}{\sqrt{6}} \frac{\kappa^2 \phi^2}{4\pi} = N \sqrt{\frac{4}{3\pi}} \frac{m}{M_P} = 5 \times 10^{-5} \Rightarrow \qquad m = 1.2 \times 10^{-6} M_P \\ = 1.4 \times 10^{13} \text{ GeV} \end{aligned}$$

$$n_s &= 1 - \frac{2}{N} \approx 0.967, \qquad \frac{dn_s}{d\ln k} = \frac{2}{N^2} = 6 \times 10^{-4} \\ A_T &= \frac{4}{\sqrt{\pi}} \frac{H}{M_P} < 10^{-5}, \qquad n_T = -2\epsilon = -\frac{1}{N} \simeq -0.016 \qquad \left[r = \frac{8}{N} \simeq 0.13\right] \end{aligned}$$
Not ruled out !

STAROBINSKY INFLATION Quantum field theory in curved space time $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2 \langle T_{\mu\nu} \rangle_{\rm ren} = \frac{1}{6M^2} {}^{(1)}H_{\mu\nu}$ ${}^{(1)}H_{\mu\nu} = 2(\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\nabla^{2})R + 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^{2}$ effective action formalism, $\mathcal{S}_g = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} \left(R - \frac{R^2}{6M^2} \right) \equiv \int d^4x \sqrt{-g} \frac{1}{2\kappa^2} f(R)$ conformal transformation $\tilde{g}_{\mu\nu} = f'(R) g_{\mu\nu} \equiv e^{\alpha\kappa\phi} g_{\mu\nu}$ $\tilde{R} = e^{-\alpha\kappa\phi} \left| R - 3\alpha\kappa\nabla^2\phi - \frac{3}{2}\alpha^2\kappa^2(\partial\phi)^2 \right| \qquad \alpha^2 = 2/3$ $V(\phi) = \frac{1}{2\kappa^2} \frac{f(R) - Rf'(R)}{(f'(R))^2} = \frac{R^2}{12\kappa^2 M^2} \left(1 - \frac{R}{3M^2}\right)^{-2}$

$$\begin{aligned} & \text{STAROBINSKY INFLATION} \\ V(\phi) &= \frac{3M^2}{4\kappa^2} \left(1 - e^{-\alpha\kappa\phi} \right)^2 = \frac{1}{2}M^2\phi^2 \left(1 + \alpha\kappa\phi + \dots \right) \\ \epsilon &= \frac{2\alpha^2}{(e^{\alpha\kappa\phi} - 1)^2} = 1 \quad \Rightarrow \quad \phi_{\text{end}} = \frac{\sqrt{3}M_P}{4\sqrt{\pi}} \ln\left(1 + \frac{2}{\sqrt{3}} \right) \simeq \frac{M_P}{5.33} \\ \eta &= \frac{2\alpha^2(2 - e^{\alpha\kappa\phi})}{(e^{\alpha\kappa\phi} - 1)^2} \quad \xi = \frac{4\alpha^4(e^{\alpha\kappa\phi} - 4)}{(e^{\alpha\kappa\phi} - 1)^3} \quad \epsilon_{60} \simeq \frac{3}{4N^2}, \ \eta_{60} \simeq -\frac{1}{N} \\ N &= \frac{e^{\alpha\kappa\phi} - \alpha\kappa\phi}{2\alpha^2} \Big|_{\phi_{\text{end}}}^{\phi} \simeq \frac{3}{4}e^{\alpha\kappa\phi} \quad \Rightarrow \quad \phi_{60} = 1.09M_P \quad \xi_{60} \simeq \frac{1}{N^2} \\ A_S &= \frac{\alpha N}{2\pi}\kappa H = 5 \times 10^{-5} \quad \Rightarrow \quad M \simeq 2.4 \times 10^{-6}M_P \\ \hline n_s = 1 - \frac{2}{N} \approx 0.967 \\ M_T &= -\frac{3}{2N^2} = -2.8 \times 10^{-4} \quad A_T = \frac{\sqrt{2}}{\pi} \frac{H}{M_P} = 2.7 \times 10^{-6} \quad reled out ! \end{aligned}$$

INFLATIONARY MODEL BUILDING



Predictions of Inflation



Cosmic Microwave Background





Discovery of CMB

Arno Penzias Robert Wilson (1965)









COBE 4-year Measurements (1992-1996)

First Measurements Temperature Anisotropies (1992)

 $\frac{\Delta T}{T_0} \approx 10^{-5}$



"for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation"





John C. Mather



CMB **lemperature** Anisotropies

The microwave background is a snapshot of the last scattering surface



The anisotropies reflect the perturbations in the surface of last scattering

gravity

Photon-Baryon Plasma in equilibrium: Thomson Scattering velocity

Metric Perturbation

density







CMB TEMPERATURE ANISOTROP |FS|gravity + density + velocity $\frac{\delta T}{T}(\mathbf{r}) = \Phi(\mathbf{r}, t_{\text{dec}}) + 2 \int_{t_{\text{dec}}}^{t_0} \dot{\Phi}(\mathbf{r}, t) dt + \frac{1}{3} \frac{\delta \rho}{\rho}(\mathbf{r}, t_{\text{dec}}) - \frac{\mathbf{r} \cdot \mathbf{v}}{c}$ The Sachs-Wolfe effect on large angular scales $\delta \rho / \rho = -2\Phi$ (for adiabatic perturbations) $\frac{\delta T}{T}(\theta,\phi) = \frac{1}{3}\Phi(\eta_{\rm LS}) Q(\eta_0,\theta,\phi) + 2 \int_{-\pi}^{\prime\prime 0} dr \,\Phi'(\eta_0-r) Q(r,\theta,\phi)$ $Q(\mathbf{x})$ eigenfunctions of the Laplacian $\Phi(\eta, \mathbf{x}) \equiv \Phi(\eta) Q(\mathbf{x})$ $\nabla^2 Q_{klm}(r,\theta,\phi) = -k^2 Q_{klm}(r,\theta,\phi)$ $Q_{klm}(r,\theta,\phi) = \Pi_{kl}(r) Y_{lm}(\theta,\phi) \qquad \Pi_{kl}(r) = \sqrt{\frac{2}{\pi}} k j_l(kr)$ $\Phi'' + 3\mathcal{H} \Phi' + a^2 \Lambda \Phi - 2K \Phi = 0$





SACHS-WOLFE PLATEAU

$$\begin{split} \frac{\delta T}{T}(\theta,\phi) &= \frac{1}{3} \Phi(\eta_{\rm LS}) \, Q = \frac{1}{5} \mathcal{R} \, Q(\eta_0,\theta,\phi) \equiv \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi) \\ \text{two-point correlation function} & C_l = \langle |a_{lm}|^2 \rangle \\ C(\theta) &= \left\langle \frac{\delta T}{T}^*(\mathbf{n}) \frac{\delta T}{T}(\mathbf{n}') \right\rangle_{\mathbf{n}\cdot\mathbf{n}'=\cos\theta} = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) \, C_l \, P_l(\cos\theta) \\ C_l^{(S)} &= \frac{4\pi}{25} \int_0^\infty \frac{dk}{k} \, \mathcal{P}_{\mathcal{R}}(k) \, j_l^2(k\eta_0) & \mathcal{P}_{\mathcal{R}}(k) = A_S^2(k\eta_0)^{n-1} \\ C_l^{(S)} &= \frac{2\pi}{25} \, A_S^2 \, \frac{\Gamma[\frac{3}{2}] \, \Gamma[1 - \frac{n-1}{2}] \, \Gamma[l + \frac{n-1}{2}]}{\Gamma[\frac{3}{2} - \frac{n-1}{2}] \, \Gamma[l + 2 - \frac{n-1}{2}]} \\ \frac{l(l+1) \, C_l^{(S)}}{2\pi} = \frac{A_S^2}{25} = \text{ constant , for } n = 1 \end{split}$$

SACHS-WOLFE PLATEAU

gauge-invariant tensor perturbation

$$h_k'' + 2\mathcal{H} h_k' + (k^2 + 2K) h_k = 0 \implies h_k(\eta) = 3h j_1(k\eta)/k\eta$$
$$\frac{\delta T}{T}(\theta, \phi) = \int_{\eta_{\rm LS}}^{\eta_0} dr \, h'(\eta_0 - r) \, Q_{rr}(r, \theta, \phi)$$

 Q_{rr} is rr-component of tensor harmonic along line of sight

$$Q_{kl}^{rr}(r) = \left[\frac{(l-1)l(l+1)(l+2)}{\pi k^2}\right]^{1/2} \frac{j_l(kr)}{r^2} \qquad I_{kl} = \int_0^{x_0} dx \frac{j_2(x_0-x)j_l(x)}{(x_0-x)x^2}$$
$$C_l^{(T)} = \frac{9\pi}{4} (l-1)l(l+1)(l+2) \int_0^\infty \frac{dk}{k} \mathcal{P}_g(k) I_{kl}^2 \qquad \mathcal{P}_g(k) = A_T^2 (k\eta_0)^{n_T}$$

$$l(l+1) C_l^{(T)} = \frac{\pi}{36} \left(1 + \frac{48\pi^2}{385} \right) A_T^2 B_l \quad \text{for } n_T = 0$$

 $B_l = (1.1184, 0.8789, \dots, 1.00)$ for $l = 2, 3, \dots, 30$


Wikinson **MCLOMANE** Anisotropy Probe





Wilkinson Microwave Anisotropy Probe (2003)













WMAP-5yr (2009)



Gaussian spectrum



WMAP-9yr (2013)





Planck (2013)

Planck Power Spectrum



GEOMETRY OF THE UNIVERSE













OPEN



CLOSED

 $\Omega_{K} = |\Omega_{0} - 1| < 0.0009 \pm 0.0056$ Planck (2013)



Spatial Curvature



geodesics

CMB Polarization Anisotropies





how we see it ...





Light carries energy and polarization (vector field)

A vector field has two comp. gradient + curl (E + B)

Linear Polarization of CMB



Polarization around Hot spots





E Polarization **B** Polarization $\nabla \times E = 0$ $\nabla \cdot B = 0$

Polarization around Cold spots





E Polarization **B** Polarization $\nabla \times E = 0$ $\nabla \cdot B = 0$

Simulations

Scalar+Tensor Perturbations 42' beam, 30deg. diam. polar cap



Simulations

Tensor Perturbations

42' beam, 30deg. diam. polar cap







Scott-Amundsen South Pole Station







Frequency Coverage of different Exp.



Measurements BICEP2



Total Polarization



B-mode Contribution



B-mode Contribution


Measurements BICEP2

BICEP2 B-mode signal



Measurements BICEP2



Measurements BICEP2





COUC Panck confirm it?



Planck TT @ intermediate scales





Could Planck confirm BICEP results?

- Some tension Planck: r < 0.11 at 95% c.l.
- Assumes 6-parameter ACDM model
- Degeneracies between parameters
- If add running tilt then tensors are relaxed



Planck sensitivity in LFI & HFI



Other Experiments coming on line

South Pole CMB telescopes



Chile(Atacama): PolarBear, ACTpol Baloon @ South Pole: EBEX(6K)

Future experiments that may confirm BICEP



Future experiments that may confirm BICEP



EBEX 6K



What is the energy scale of nfation?



 n_s

What is the energy scale of inflation?





Inflation and fundamental physics



Structure Formation





$z \approx 1100$ CMB Anisotropies

 $z \approx 100$ Dark ages

 $z \approx 20$ First stars

 $z \approx 10$ Galaxies & Quasars

 $z \approx 1$

Clusters & Superclusters

First Stars and Reionization Era











Predictions of Inflation



The Global structure of the universe







