Redshift-space distortions: two-point correlation function in wide-angle regime

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II Azores School on Observational Cosmology

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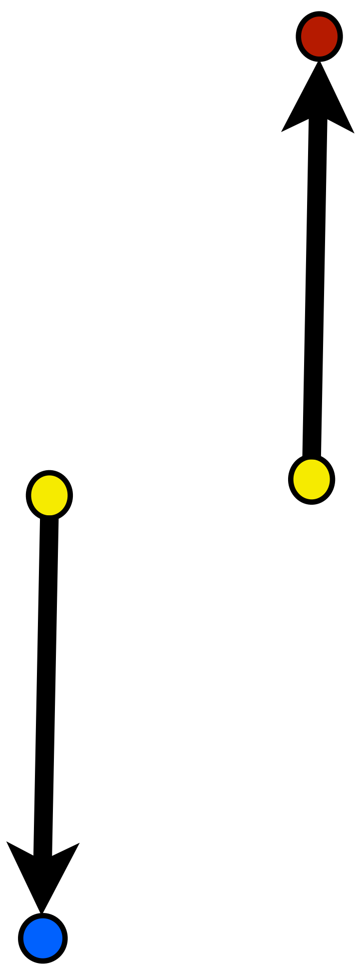
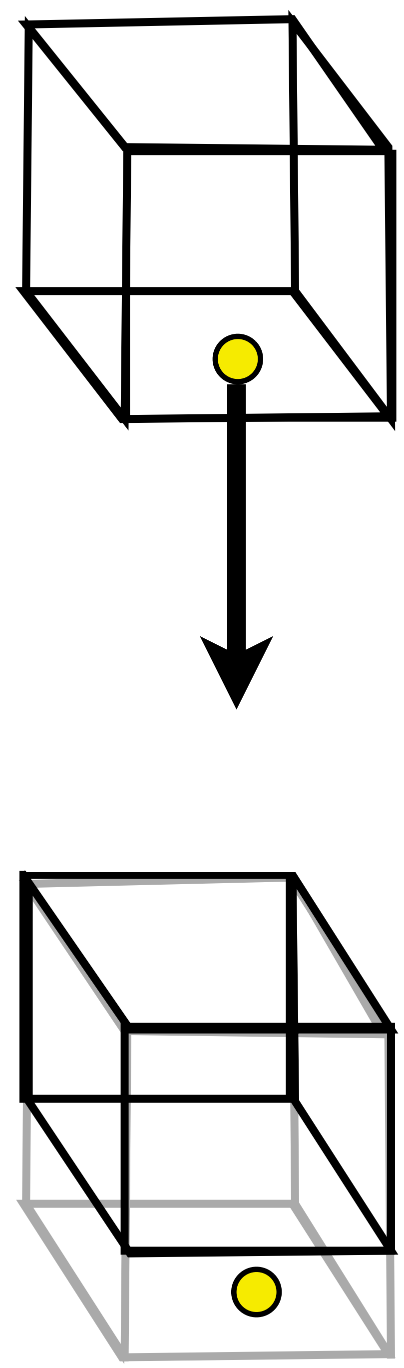
SEVENTH FRAMEWORK PROGRAMME

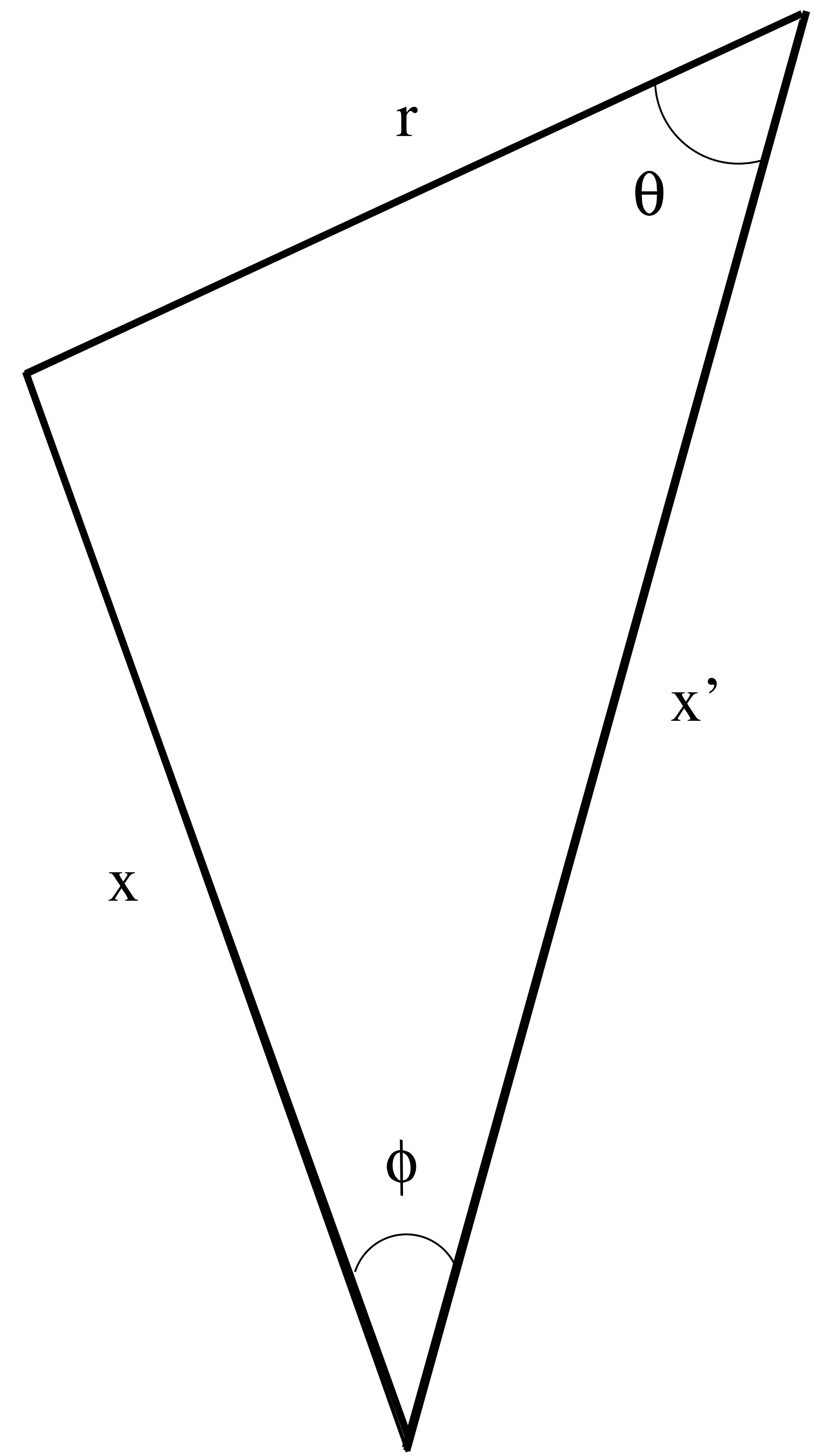


$$\mathbf{s} = \mathbf{x} - (\mathbf{v} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}}$$



$$\delta(\mathbf{s}) = \delta(\mathbf{x}) - \partial_r(\mathbf{u} \cdot \hat{\mathbf{x}}) + \frac{2}{x}\mathbf{u} \cdot \hat{\mathbf{x}}$$

Bonvin & Durrer 11, Challinor & Lewis 11, Yoo et al. 09, Jeong et al. 12, Bertacca et al. 12



$\nu = \cos\phi$ $\mu = \cos\theta$

From Pápai & Szapudi (2008):

$$\xi_s(\phi_1,\phi_2,r) = \sum_{n_1,n_2=0,1,2} a_{n_1n_2}\cos(n_1\phi_1)\cos(n_2\phi_2) + b_{n_1n_2}\sin(n_1\phi_1)\sin(n_2\phi_2).$$

Again, for reference, the previously calculated coefficients are

$$a_{00} = \left(1+\frac{2f}{3}+\frac{2f^2}{15}\right)\xi_0^2(r) - \left(\frac{f}{3}+\frac{2f^2}{21}\right)\xi_2^2(r) + \frac{3f^2}{140}\xi_4^2(r),$$

$$a_{02} = a_{20} = \left(\frac{-f}{2}-\frac{3f^2}{14}\right)\xi_2^2(r) + \frac{f^2}{28}\xi_4^2(r),$$

$$a_{22} = \frac{f^2}{15}\xi_0^2(r) - \frac{f^2}{21}\xi_2^2(r) + \frac{19f^2}{140}\xi_4^2(r),$$

$$b_{22} = \frac{f^2}{15}\xi_0^2(r) - \frac{f^2}{21}\xi_2^2(r) - \frac{4f^2}{35}\xi_4^2(r);$$

and the new expressions of this work correspond to

$$a_{10} = \frac{\tilde{a}_{10}}{g_1} = \left(2f+\frac{4f^2}{5}\right)\frac{1}{g_1 r}\xi_1^1 - \frac{1}{5}\frac{f^2}{g_1 r}\xi_3^1,$$

$$a_{01} = \frac{\tilde{a}_{01}}{g_2} = -\left(2f+\frac{4f^2}{5}\right)\frac{1}{g_2 r}\xi_1^1 + \frac{1}{5}\frac{f^2}{g_2 r}\xi_3^1,$$

$$a_{11} = \frac{\tilde{a}_{11}}{g_1 g_2} = \frac{4}{3}\frac{f^2}{g_1 g_2 r^2}\xi_0^0 - \frac{8}{3}\frac{f^2}{g_1 g_2 r^2}\xi_2^0,$$

$$a_{21} = \frac{\tilde{a}_{21}}{g_2} = -\frac{2}{5}\frac{f^2}{g_2 r}\xi_1^1 + \frac{3}{5}\frac{f^2}{g_2 r}\xi_3^1,$$

$$a_{12} = \frac{\tilde{a}_{12}}{g_1} = \frac{2}{5}\frac{f^2}{g_1 r}\xi_1^1 - \frac{3}{5}\frac{f^2}{g_1 r}\xi_3^1,$$

$$b_{11} = \frac{\tilde{b}_{11}}{g_1 g_2} = \frac{4}{3}\frac{f^2}{g_1 g_2 r^2}\xi_0^0 + \frac{4}{3}\frac{f^2}{g_1 g_2 r^2}\xi_2^0,$$

$$b_{21} = \frac{\tilde{b}_{21}}{g_2} = -\frac{2}{5}\frac{f^2}{g_2 r}\xi_1^1 - \frac{2}{5}\frac{f^2}{g_2 r}\xi_3^1,$$

$$b_{12} = \frac{\tilde{b}_{12}}{g_1} = \frac{2}{5}\frac{f^2}{g_1 r}\xi_1^1 + \frac{2}{5}\frac{f^2}{g_1 r}\xi_3^1.$$

where

$$\xi_l^m(r) = \int dk/2\pi^2 k^m j_l(rk)P(k)$$

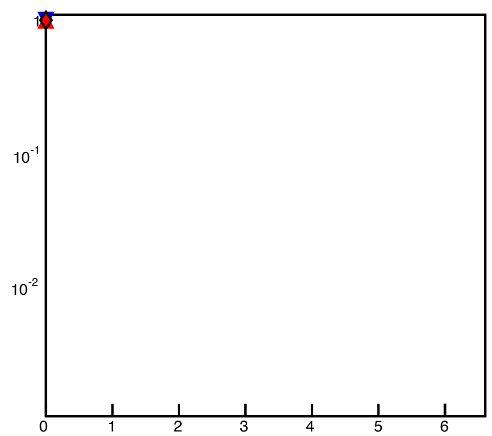
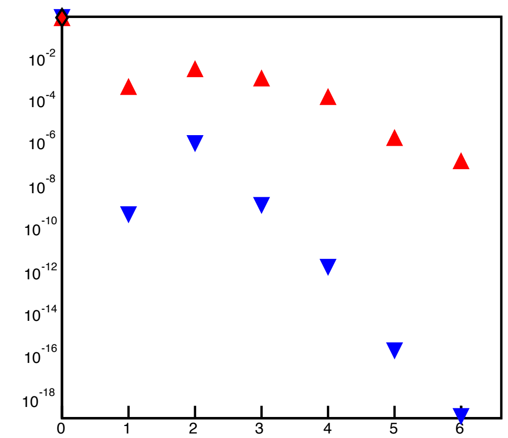
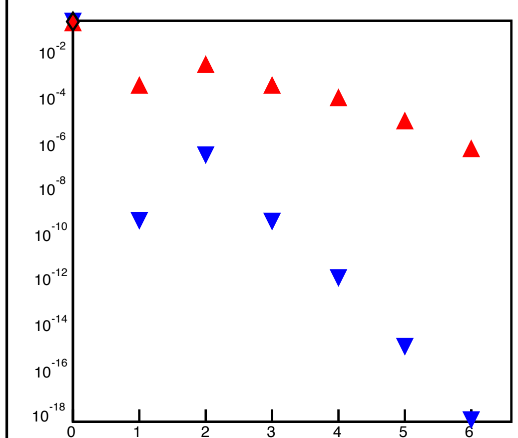
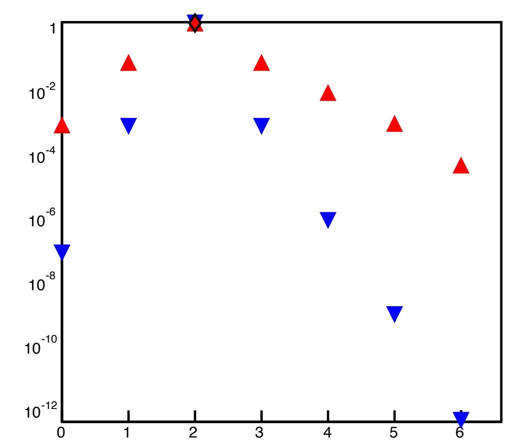
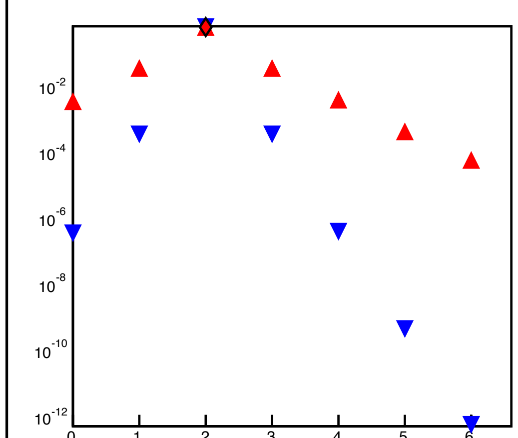
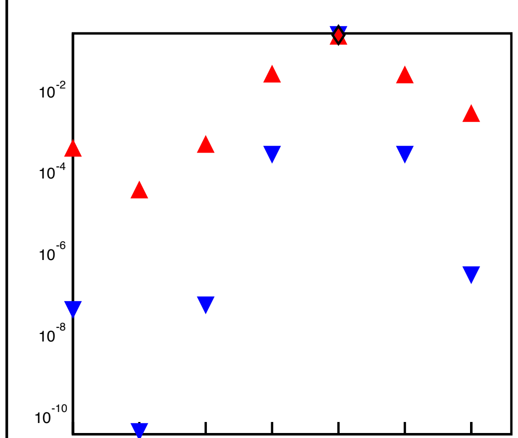
$$g_1 r = \frac{\sin(\phi_2)}{\sin(\phi_2-\phi_1)} r \qquad g_2 r = \frac{\sin(\phi_1)}{\sin(\phi_2-\phi_1)} r$$

Raccanelli et al. 2010, Montanari & Durrer 2012

Papai & Szapudi, 2008, Raccanelli et al. 2010, Montanari & Durrer 2012

r/x'

$${}^{z}\xi(r,x',\mu) = \sum_{l=0}^{\infty} {}^{z}\xi_l(r,x')\mathcal{P}_l(\mu)$$

	$P_{\delta\delta}(k)$	$P_{\delta\theta}(k)$	$P_{\theta\theta}(k)$
$\frac{1}{2\pi^2}\int dk k^2 j_0(kr)$			
$-\frac{1}{2\pi^2}\int dk k^2 j_2(kr)$			
$\frac{1}{2\pi^2}\int dk k^2 j_4(kr)$			

in small-angle

$${}^{z}\xi_0(r) = \frac{1}{2\pi^2}\int dk k^2 j_0(kr)\left(P_{\delta\delta}(k) + \frac{2}{3}P_{\delta\theta}(k) + \frac{1}{5}P_{\theta\theta}(k)\right),$$

$${}^{z}\xi_2(r) = -\frac{1}{2\pi^2}\int dk k^2 j_2(kr)\left(\frac{4}{3}P_{\delta\theta}(k) + \frac{4}{7}P_{\theta\theta}(k)\right),$$

$${}^{z}\xi_2(r) = \frac{1}{2\pi^2}\int dk k^2 j_4(kr)\frac{8}{35}P_{\theta\theta}(k).$$

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(+ one term geometrically suppressed term multiplying $\int dk\, k^2 \frac{j_0(kr)}{(kr)^2}P_{\theta\theta}(k)$)

Schematic calculation:

The two point correlation function in redshift space is given by:

$$ {}^{z}\xi(\mathbf{x},\mathbf{x}') := \langle {}^{z}\delta(\mathbf{x})\, {}^{z}\delta(\mathbf{x}')^{*} \rangle $$

where, the spherically decomposed density field is:

$$ {}^{z}\delta(\mathbf{x}) = \sum_{l,m} (-i)^{l} \sqrt{\frac{2}{\pi}} \int dk k^{2} j_{l}(kx)\, {}^{z}\delta_{lm}(k) \mathrm{Y}_{lm}(\hat{\mathbf{x}}) $$

assuming that peculiar velocities are small, first order perturbation theory allow us to write:

Heavens & Taylor 95

$$ {}^{z}\delta_{lm}(k) = \delta_{lm}(k) + \int dk' N_{l}(k,k')\theta_{lm}(k') $$

with the kernel

$$ N_{l}(k,k') = \frac{2}{\pi} \int dr r^{2} j_{l}'(k'r) j_{l}'(kr) k k' = \delta(k-k') - \frac{l(l+1)}{(2l+1)} \frac{k_{<}^{l}}{k_{>}^{l+1}} $$

As final result, the two point correlation function can be written as:

$$
\begin{aligned}
{}^{z}\xi(\mathbf{x},\mathbf{x}') &= \frac{1}{2\pi^2}\int dk k^2 j_0(kr)\left(P_{\delta\delta}(k)+2P_{\delta\theta}(k)+P_{\theta\theta}(k)\right)\\
&+ \frac{1}{2\pi^2}\frac{d}{d\nu}\left[(1-\nu^2)\frac{d}{d\nu}\right]\int dk j_0(kr)\left(P_{\delta\theta}(k)+P_{\theta\theta}(k)\right)\left(\frac{1}{x^2}+\frac{1}{x'^2}\right)\\
&+ \frac{1}{2\pi^2}\left[\frac{d}{d\nu}\left((1-\nu^2)\frac{d}{d\nu}\right)\right]^2\int dk j_0(kr)\frac{P_{\theta\theta}(k)}{k^2}\frac{1}{x^2x'^2}
\end{aligned}
$$

or, in Fourier space, we obtain:

$$
\begin{aligned}
\langle {}^{z}\delta(\mathbf{k}), {}^{z}\delta(\mathbf{k}')^{*}\rangle &= \left(P_{\delta\delta}(k)+2P_{\delta\theta}(k)+P_{\theta\theta}(k)\right)\delta(\mathbf{k}-\mathbf{k}')\\
&+\frac{1}{4\pi}\left[\left(\frac{P_{\delta\theta}(k)}{k^2}+\frac{P_{\theta\theta}(k)}{k^2}\right)+\left(\frac{P_{\delta\theta}(k')}{k'^2}+\frac{P_{\theta\theta}(k')}{k'^2}\right)\right]\frac{\partial}{\partial\gamma}\left[(1-\gamma^2)\frac{\partial}{\partial\gamma}\right]\frac{1}{|\mathbf{k}-\mathbf{k}'|}\\
&+\frac{1}{(4\pi)^2}\int d^2\hat{\mathbf{k}}''\frac{\partial}{\partial\gamma_1}\left[(1-\gamma_1^2)\frac{\partial}{\partial\gamma_1}\right]\frac{\partial}{\partial\gamma_2}\left[(1-\gamma_2^2)\frac{\partial}{\partial\gamma_2}\right]\\
&\times\int dk''\frac{1}{|\mathbf{k}-\mathbf{k}''|}\frac{P_{\theta\theta}(k'')}{k''^2}\frac{1}{|\mathbf{k}''-\mathbf{k}'|}
\end{aligned}
$$