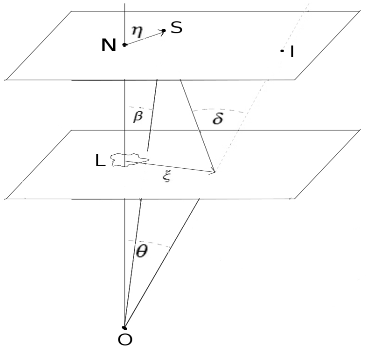
Weak gravitational lensing by rotating tidal charged black holes

Zsolt Horváth

2nd Azores School on Observational Cosmology



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Lensing concepts

- lensing: observable consequences of light deflection (and the theory of them)
  - [multiple] image formation, distortion, magnification, time delay
- note on terminology

"astronomers"

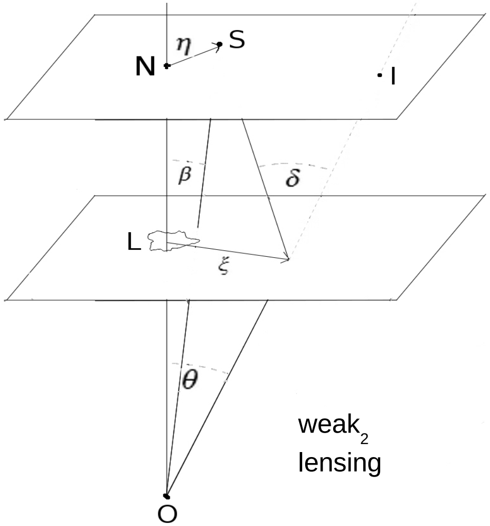
weak$_1$ lensing: slight elliptical distortion of galaxy images

strong$_1$ lensing: multiple images, arcs, Einstein rings

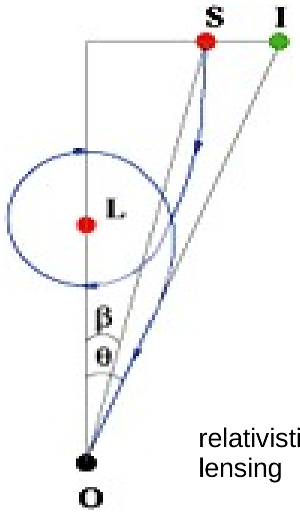
"theoreticians"

weak$_2$ lensing: small angle approximation holds for the deflection angle

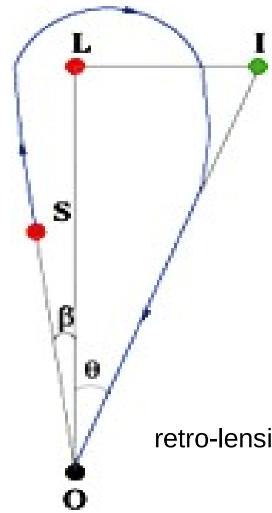
strong$_2$ lensing: the deflection angle is close to an integer multiple of π



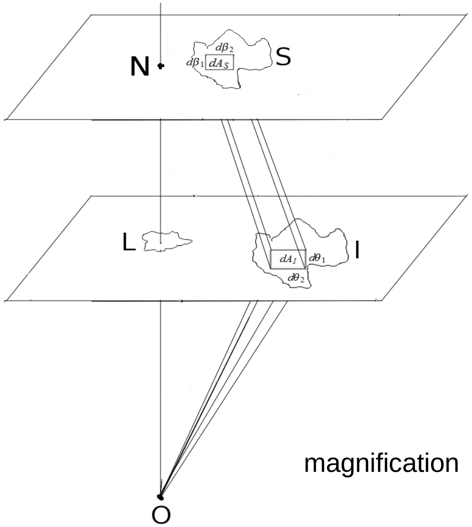
weak$_2$ lensing



relativistic lensing



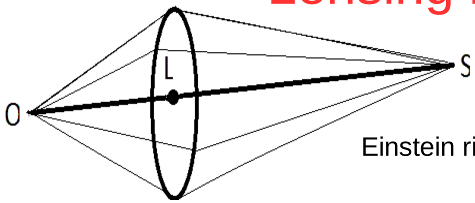
retro-lensing



magnification

- light propagates along null geodesics
- **weak lensing theory** (approximation): straight lines + Euclidean trigonometry
- let be the source and the observer in weak field

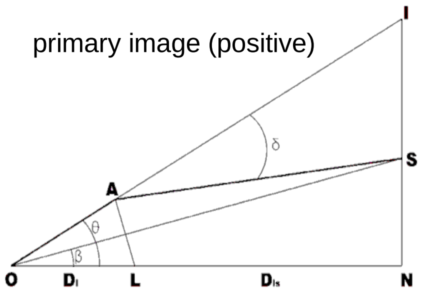
Lensing in spherically or axially symmetric spacetimes

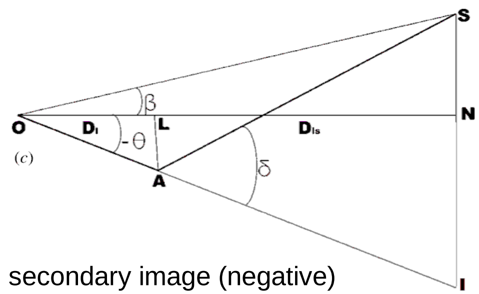


Einstein ring

- planar light propagation
- pointlike S + sign conventions

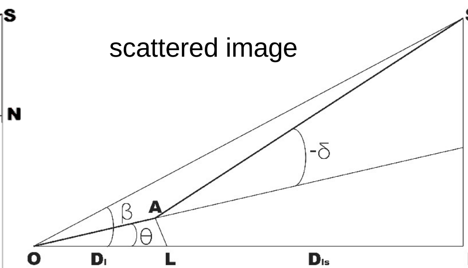
primary image (positive)





secondary image (negative)

scattered image



b.)

P. Schneider, J. Ehlers, E. E. Falco, Gravitational Lenses

K.S. Virbhadra, G.F.R. Ellis, PRD 62, 084003 (2000).

D.E. Holz, J.A. Wheeler, ApJ. 578, 330 (2002)

Lens equations

- there is a variety of lens equations, they may differ in precision
- examples:

V. Bozza, Phys. Rev. D 78, 103005 (2008).

$$\theta - \beta - \frac{D_{LS}}{D_S}\delta(\theta) = 0$$

small angles l.e.

$$\tan|\theta| - \tan(s\beta) - \frac{D_{ls}}{D_s}\left[\tan|\theta| + \tan(\delta - |\theta|)\right] = 0$$

Virbhadra-Ellis l.e

"exact" l.e.

$$0 = \frac{2D_L}{D_S}\cos\left(\frac{\delta}{2} - |\theta|\right)\cos|\theta|\sin\frac{\delta}{2} + \cos(\delta - |\theta|)\left(\sin|\theta| - s\cos|\theta|\tan\beta\right) - \sin\delta$$

- most simple application

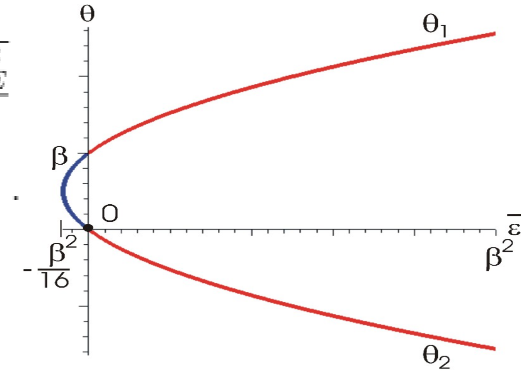
$$\delta = 4\frac{GM}{c^2 b} \qquad b = D_L|\Theta|$$

$$0 = \theta^2 - \beta\theta - 4\frac{G}{c^2}M\frac{D_{LS}}{D_L D_S}$$

$$\theta_\pm = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2}$$

$$\theta_E^2 := 4\frac{G}{c^2}M\frac{D_{LS}}{D_L D_S}.$$

"Schwarzschild lensing"



a.)

Lensing by tidal charged (non-rotating) brane black holes

- brane theory: one kind of alternative theory of gravitation
- 5D space-time in which the brane (our 4D observable universe) is embedded
- interactions act only inside the brane, except gravity
- effective Einstein equation on the brane: $G_{ab} = k_4^2 T_{ab} + k_5^4 S_{ab} - E_{ab}$
- **black hole solution** of the effective Einstein equation on the brane
- **tidal charge** q is an imprint of the Weyl curvature of the 5D spacetime

$$ds^2 = -f(r)\,dt^2 + f^{-1}(r)\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$f = 1 - \frac{2m}{r} + \frac{q}{r^2}.$$

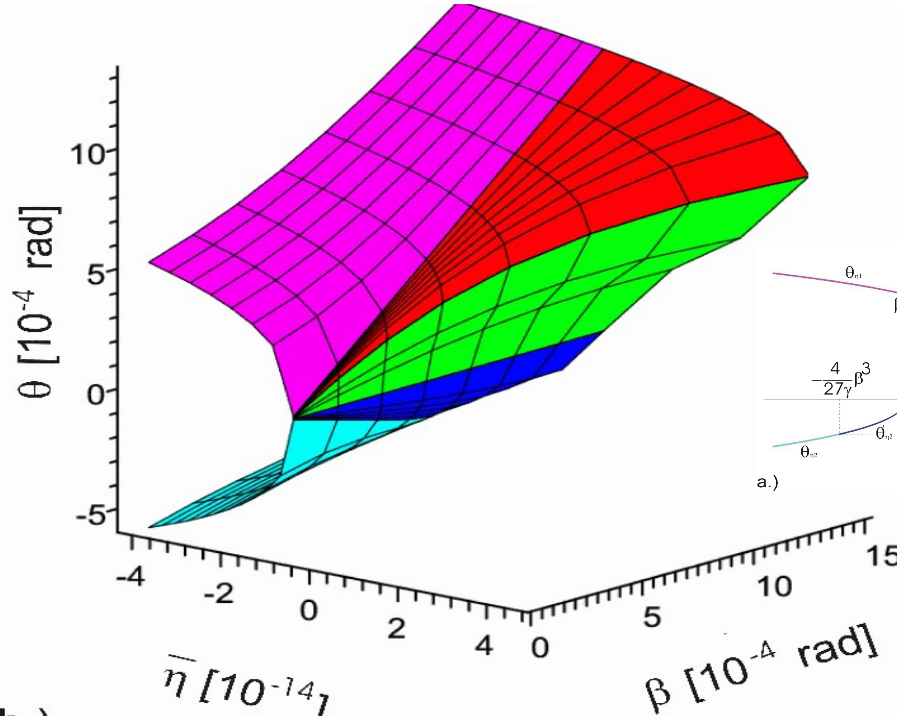
$$\varepsilon = \frac{m}{b} \qquad \eta = \frac{q}{b^2}$$

$$\delta = 4\varepsilon - \frac{3}{4}\pi\eta + \frac{15}{4}\pi\varepsilon^2 - 16\varepsilon\eta + \frac{105}{64}\pi\eta^2 \longrightarrow 0 = \theta^3 - \beta\theta^2 - 4\bar{\varepsilon}\theta + s\gamma\left(\bar{\eta} - 5\bar{\varepsilon}^2\right)$$

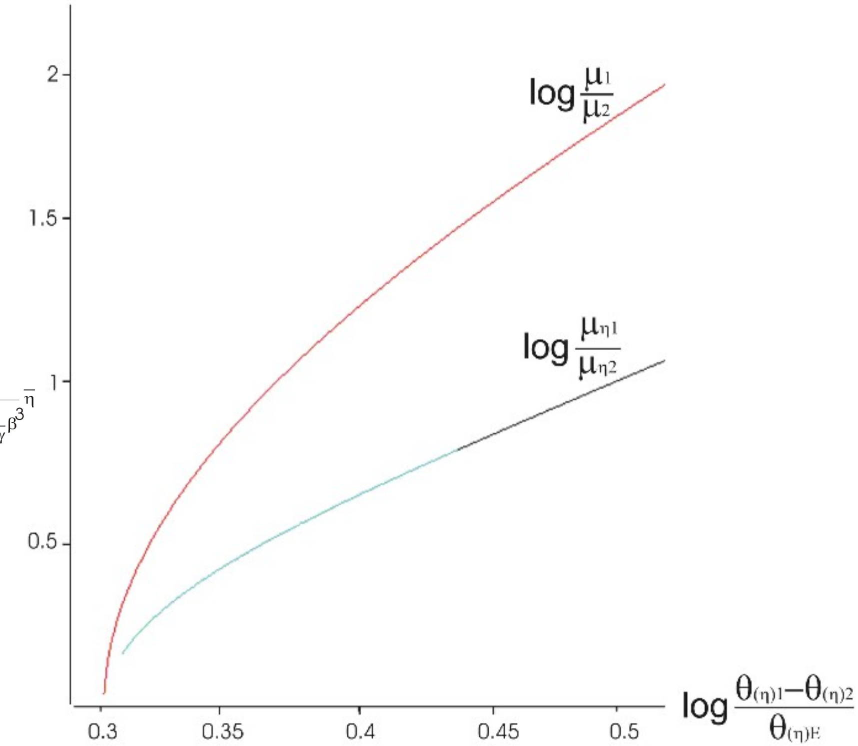
$$\bar{\varepsilon} = \frac{m D_{LS}}{D_L D_S} \qquad \bar{\eta} = \frac{q D_{LS}}{D_L D_S}$$

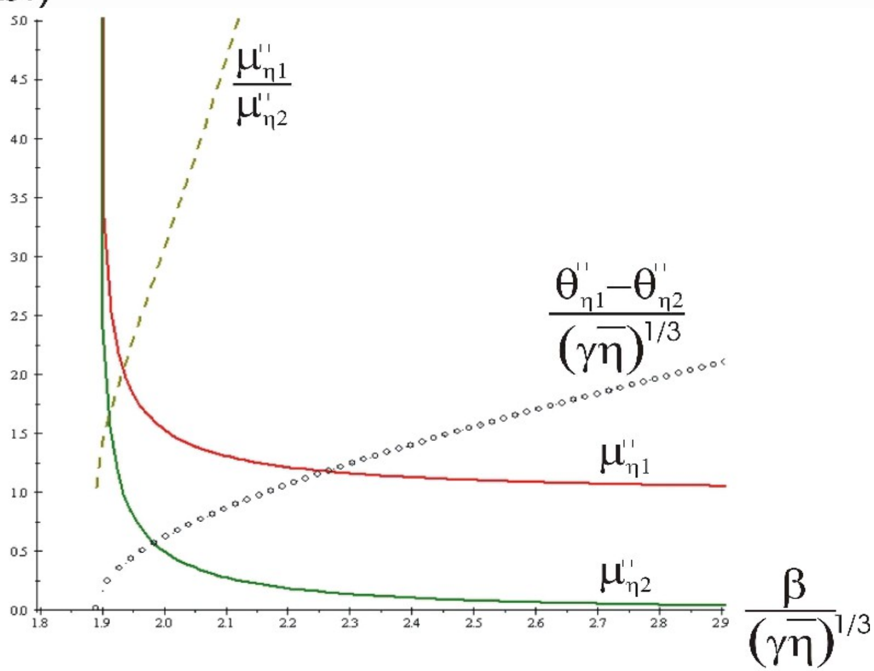
L. Á. Gergely, Z. Keresztes, M. Dwornik, Class. Quant. Grav. 26, 145002 (2009).

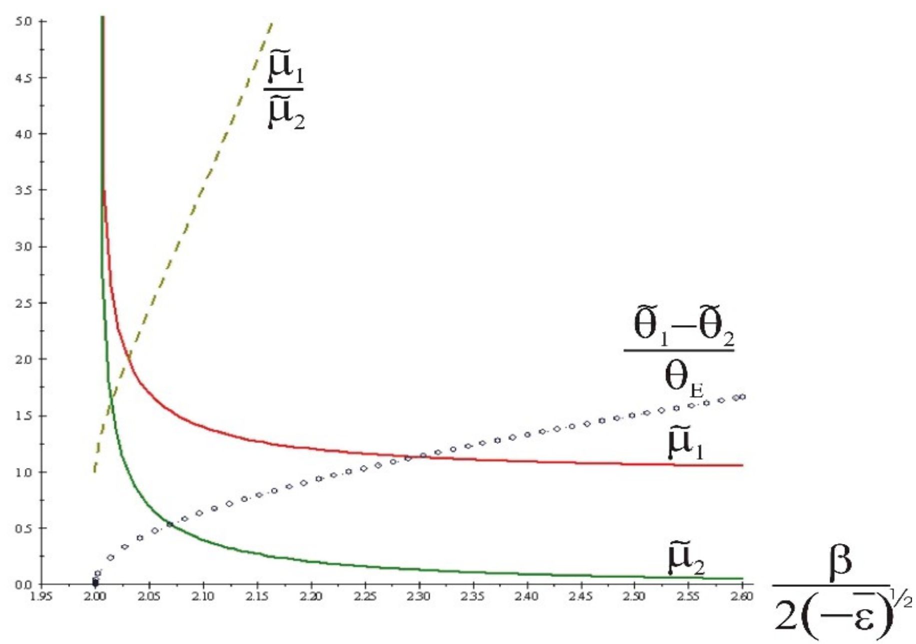
Z. Horváth, L. Á. Gergely, D. Hobill, Class. Quant. Grav. 27, 235006 (2010).


θ [10^-4 rad]
η̄ [10^-14]
β [10^-4 rad]
a.)

b.)


log μ1/μ2
log μη1/μη2
log (θ(η)1−θ(η)2)/θ(η)E


μ''η1/μ''η2
(θ''η1−θ''η2)/(γη̄)^1/3
μ''η1
μ''η2
β/(γη̄)^1/3


μ̃1/μ̃2
(θ̃1−θ̃2)/θE
μ̃1
μ̃2
β/2(−ε̄)^1/2

a.)

b.)

Lensing by tidal charged rotating black holes

black hole solution: mass M, **tidal charge q, angular momentum J**

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\theta}{\Sigma}\left[adt - \left(r^2 + a^2\right)d\phi\right]^2 ,$$

a=J/M rotation parameter

$$\Delta = r\left(r - 2M\right) + a^2 + q \; , \Sigma = r^2 + a^2\cos^2\theta \; .$$

formally Kerr-Newman metric

$a \leqslant \sqrt{M^2 - q}$

$r_{horizon} = M + \sqrt{M^2 - q - a^2}$

small parameters

$$\varepsilon = \frac{M}{b} \; , \; \alpha = \frac{a}{b}, \; \eta \; = \frac{q}{b^2}$$

deflection angle

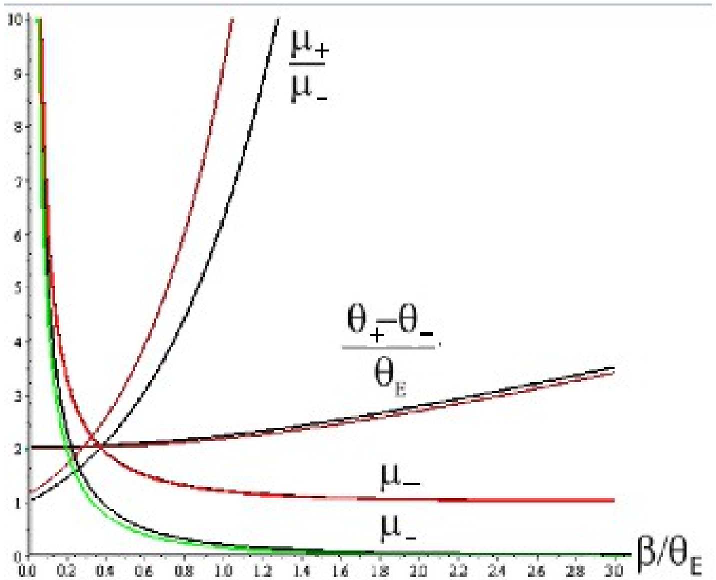
$$\delta = 4\varepsilon + \frac{15\pi}{4}\varepsilon^2 - 4\kappa\varepsilon\alpha - \frac{3\pi}{4}\eta$$

algebraic lens equation

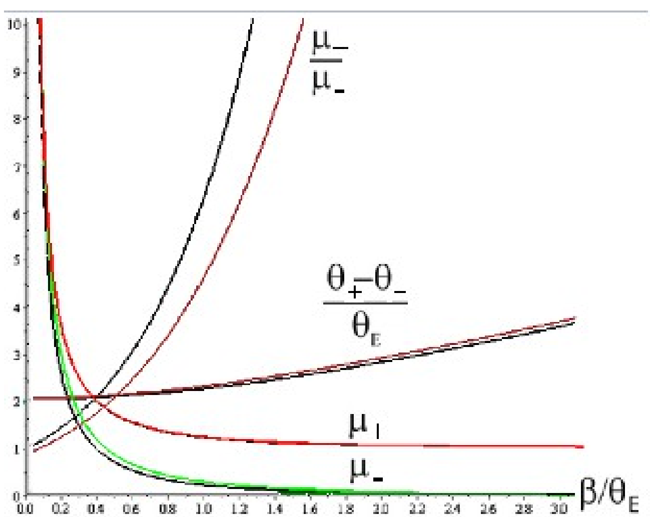
$$0 = \theta^3 - \beta\theta^2 - \theta_E^2\theta + s\gamma\left(\bar{\eta} + \frac{16}{3\pi}\kappa\bar{\varepsilon}\bar{\alpha} - 5\bar{\varepsilon}^2\right)$$

we set $a = \sqrt{M^2 - q}$ $\bar{\varepsilon} = 0.01$ $q = 5 \cdot M^2$ $D_L = 10^9$ parsec $D_L = 2 * 10^9$ parsec

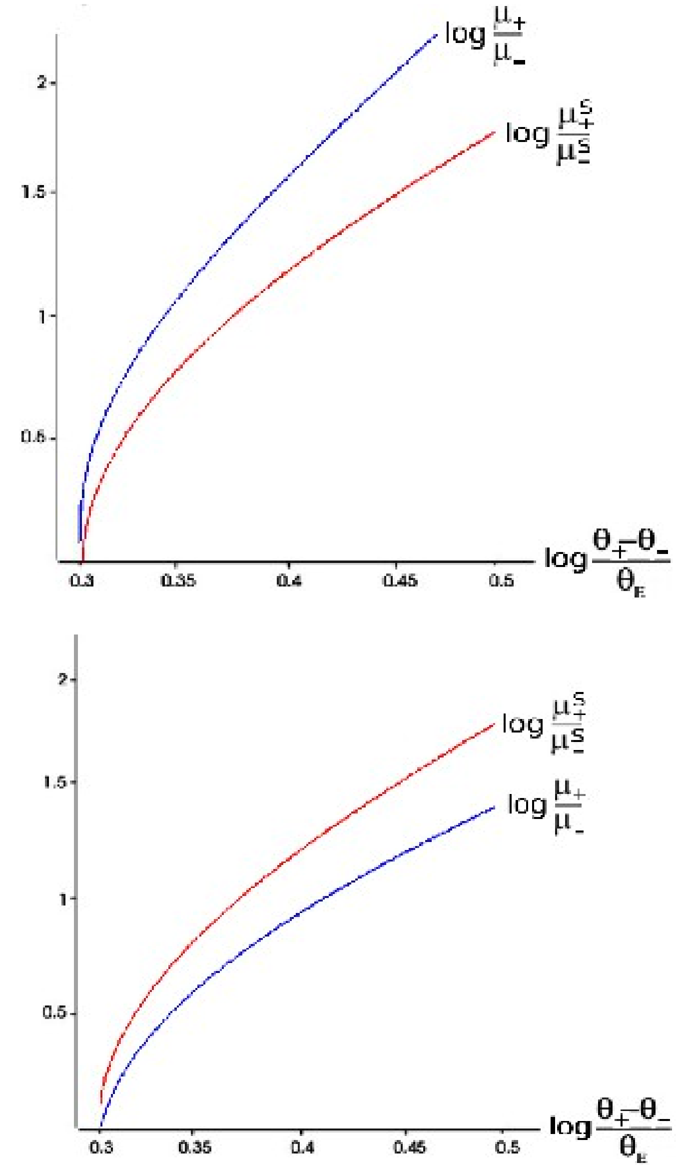
Z. Horváth, L. Á. Gergely, Weak gravitational … (in preparation)



The image separations, the magnifications of the images and their ratio, when the second order perturbations due to the mass and tidal charge cancel, in the subcase the retrograde image has negative apparent angle, the prograde image has a positive one ($\kappa \cdot s = +1$).



The image separations, the magnifications of the images and their ratio, when the second order perturbations due to the mass and tidal charge cancel, in the subcase the prograde image has negative apparent angle, the retrograde image has a positive one ($\kappa \cdot s = -1$).



The logarithm of the ratio of the primary magnification to the secondary magnification as a function of the logarithm of the image separation divided by Einstein angle.