GRAnada COde for the resolution of the adiabatic and non-adiabatic stellar oscillations

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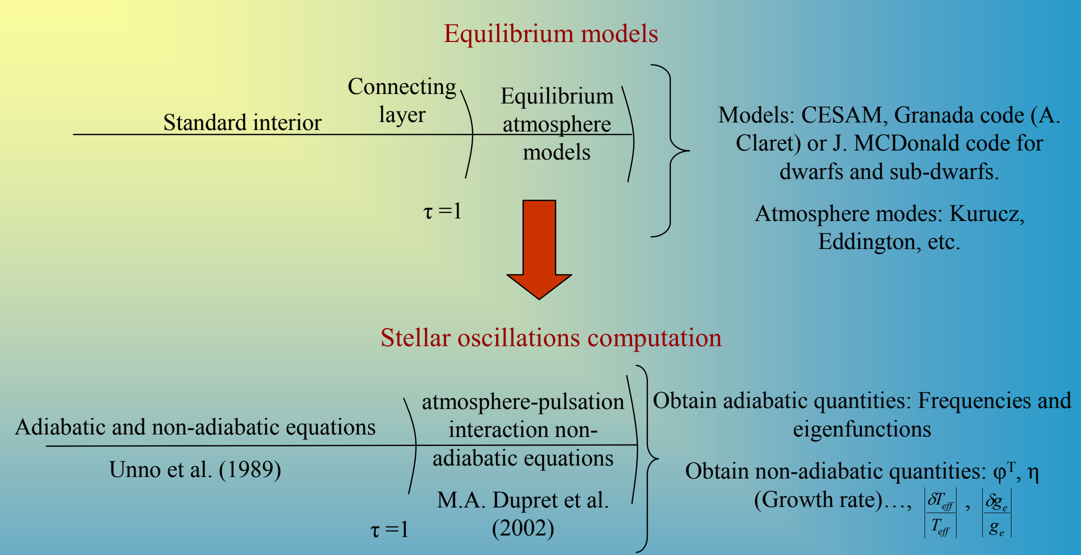
General scheme



We use the Henyey relaxation method described in Unno et al.

Perturbative equations in the interior and the atmosphere

Adiabatic resolution

The complete star solved with the same equations (Unno et al.)

$$\frac{dy_1}{d\ln x} = \left(V_g - 3\right)y_1 + \left(\frac{\lambda(\lambda+1)}{C_1\omega^2} - V_g\right) + V_g y_3$$

$$\frac{dy_2}{d\ln x} = \left(C_1\omega^2 - A^*\right)y_1 + \left(A^* - U + 1\right)y_2 - A^* y_3$$

$$\frac{dy_3}{d\ln x} = \left(1 - U\right)y_3 + y_4$$

$$\frac{dy_4}{d\ln x} = UA^* y_1 + UV_g y_2 + \left[\lambda(\lambda+1) - UV_g\right]y_3 - Uy_4$$

Perturbative equations in the interior and the atmosphere

Adiabatic resolution

With the boundary conditions

$C_1\omega^2 y_1 - \lambda y_2 = 0$ and $\lambda y_3 - y_4 = 0$ in r=0

$y_1 - y_2 + y_3 = 0$

$\frac{\lambda - b_{11}}{b_{12}} y_1 - y_2 + \left[\alpha_1 \frac{\lambda - b_{11}}{b_{12}} - \alpha_2\right] y_3 = 0$ and $\left(\lambda+1\right)y_3 + y_4 = 0$ in r=R

$$y_1 = \frac{\xi_r}{r};\, y_2 = \frac{1}{gr}\left(\frac{P'}{\rho} + \Phi'\right) = \frac{\sigma^2 \xi_h}{g};\, y_3 = \frac{1}{gr}\Phi';\, y_4 = \frac{1}{g}\frac{d\Phi'}{dr}$$

Perturbative equations in the interior and the atmosphere

Adiabatic resolution

For radial resolution we can use LAWE or ℓ=0 in these equations

Perturbative equations in the interior and the atmosphere

Non-adiabatic resolution

We can divide the star in two zones :

1) Interior: Main part of the star. Here we follow the adiabatic and non-adiabatic equations showed in Unno et al (89).

Assumptions:

1. No rotation
2. No magnetic fields
3. Diffusion approximation for the radiative flux
4. Frozen convection

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$$\frac{dy_1}{d\ln x} = (V_g - 3)y_1 + \left(\frac{\lambda(\lambda+1)}{C_1\omega^2} - V_g\right) + V_g y_3 \ + \upsilon_t y_5$$

$$\frac{dy_2}{d\ln x} = (C_1\omega^2 - A^*)y_1 + (A^* - U + 1)y_2 - A^* y_3 \ + \upsilon_t y_5$$

$$\frac{dy_3}{d\ln x} = (1 - U)y_3 + y_4$$

$$\frac{dy_4}{d\ln x} = UA^* y_1 + UV_g y_2 + [\lambda(\lambda+1) - UV_g]y_3 - Uy_4 + \upsilon_t y_5$$

$$\frac{dy_5}{d\ln x} = V\left(\nabla_{ad}(U - C_1\omega^2) - 4(\nabla_{ad} - \nabla) + C_2\right)y_1 + V\left(\frac{\lambda(\lambda+1)}{C_1\omega^2}(\nabla_{ad} - \nabla) + C_2\right)y_2 + VC_2 y_3 + V\nabla_{ad} y_4 + V\nabla(4 - \kappa_s)y_5 - V\nabla y_6$$

$$\frac{dy_6}{d\ln x} = \left(\lambda(\lambda+1)\frac{\nabla_{ad} - \nabla}{\nabla} - C_3\varepsilon_{ad}V\right)y_1 + \left(C_3\varepsilon_{ad}V - \lambda(\lambda+1)\left(\frac{\nabla_{ad}}{\nabla} - \frac{C_3}{C_1\omega^2}\right)\right)y_2 + \left(\lambda(\lambda+1)\frac{\nabla_{ad}}{\nabla} - C_3\varepsilon_{ad}V\right)y_3 + \left(C_3\varepsilon_s - \frac{\lambda(\lambda+1)}{V\nabla} - iC_4\omega\right)y_5 - \frac{d\ln L_R}{d\ln r}y_6$$

Perturbative equations in the interior and the atmosphere

With the boundary conditions

$C_1\omega^2 y_1 - \lambda y_2 = 0$, $\lambda y_3 - y_4 = 0$ and $y_5 = 0$ in r=0

$y_1 - y_2 + y_3 = 0$ or $\left[1 + \frac{\frac{\lambda(\lambda+1)}{\omega^2} - 4 - \omega^2}{V}\right]y_1 - y_2 + \left[1 + \frac{\frac{\lambda(\lambda+1)}{\omega^2} - \lambda - 1}{V}\right]y_3 = 0$

$(\lambda+1)y_3 + y_4 = 0$

$(2 - 4\nabla_{ad}V)y_1 + 4\nabla_{ad}V(y_2 - y_3) + 4y_5 - y_6 = 0$

in r=R

$$y_1 = \frac{\xi_r}{r};\ y_2 = \frac{1}{gr}\left(\frac{P'}{\rho} + \Phi'\right) = \frac{\sigma^2\xi_h}{g};\ y_3 = \frac{1}{gr}\Phi';\ y_4 = \frac{1}{g}\frac{d\Phi'}{dr};\ y_5 = \frac{\delta S}{C_p};\ y_6 = \frac{\delta L_R}{L_R}$$

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Perturbative equations in the interior and the atmosphere

2) Atmosphere: Only in non-adiabatic calculations. The external par of the star, where the photosphere is. We follow Dupret et al (02).

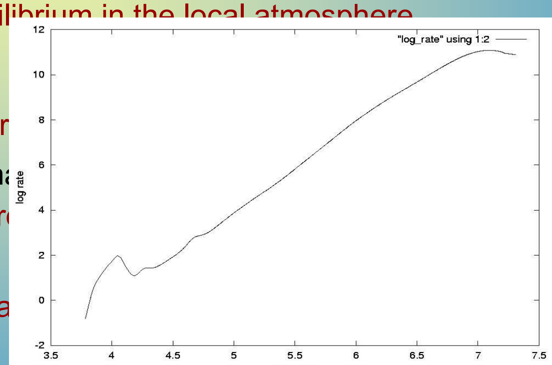
Assumptions:

1. No rotation
2. No magnetic fields
3. Plane-parallel atmosphere.
4. Frozen convection
5. Radiative equilibrium

Perturbative equations in the interior and the atmosphere

2) Stellar atmosphere: Assumptions:

- Radiative equilibrium in the local atmosphere



→ Temper... $\frac{\partial \ln T}{\partial \ln g}\frac{\delta g_e}{g_e} + \frac{\partial \ln T}{\partial \ln \tau}\frac{\delta \tau}{\tau}$

Therma... od

→ Monocr... $\frac{\partial \ln F_\lambda^+}{\partial \ln g}\frac{\delta g_e}{g_e}$ g)

→ Limb da... $\frac{\partial \ln h_\lambda}{\partial \ln g}\frac{\delta g_e}{g_e} + \frac{\partial \ln h_\lambda}{\partial \ln \mu}\frac{\delta \mu}{\mu}$

- No rotation-pulsation interaction → Spherical symmetry at equilibrium
- Frozen convection → Spherical harmonics

Perturbative equations in the interior and the atmosphere

Conecting layer:

Is the transition layer between both descriptions.

Impose continuity at a given optical depth

External boundary conditions "with" atmosphere:

$$\frac{\partial\left(\frac{\delta P_g}{P_g}\right)}{\partial r} = 0$$

$$\frac{d\phi'}{dr} + \frac{l+1}{r}\phi' = 0$$

$$\lim_{\tau \to 0} \frac{\delta\tau}{\tau} = \frac{\partial\,\delta\tau}{\partial\,\tau}$$

Non-adiabatic observables

We obtain

$$\left|\frac{\delta T_{eff}}{T_{eff}}\right| \qquad \left|\frac{\delta g_e}{g_e}\right| \qquad \varphi^T = \varphi\left(\frac{\delta T_{eff}}{T_{eff}}\right) - \varphi\left(\frac{\xi_r}{R}\right)$$

And the growth rate

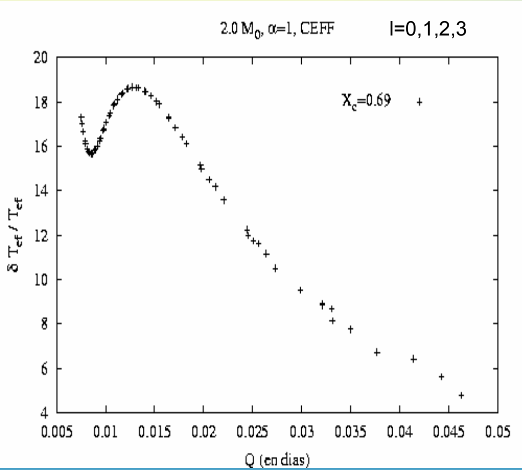
$$\eta = \frac{\oint\int_0^M W dM_r}{\oint\int_0^M |W| dM_r}$$

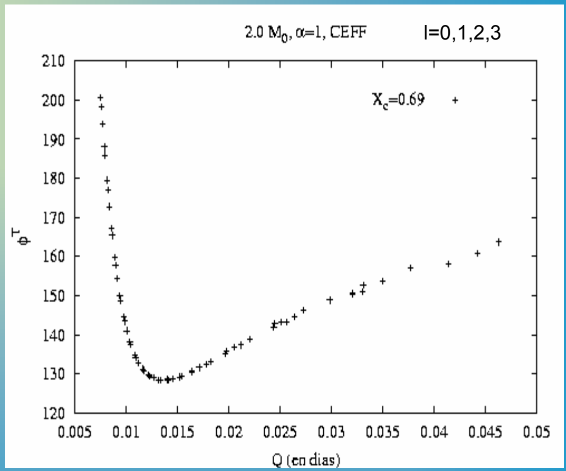
Where

$$W = \frac{\delta T}{T}\left(\delta\varepsilon_N - \nabla\cdot\vec{F}\right)$$

Results

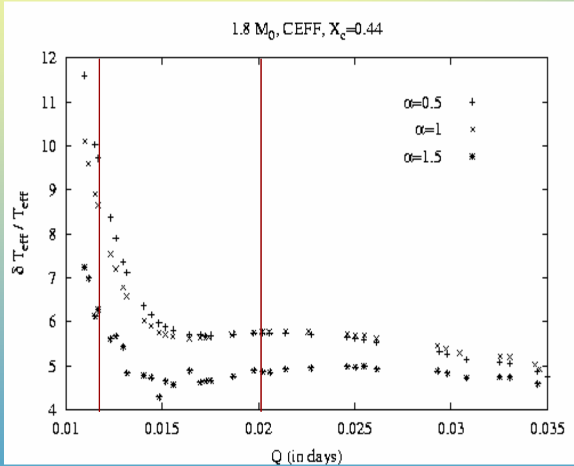
1- Non-adiabatic observables

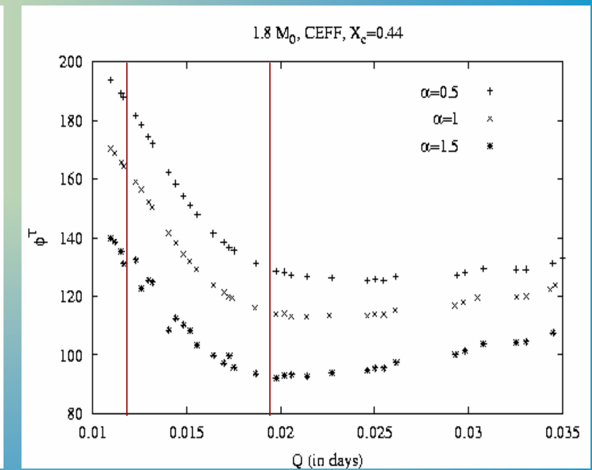




Results

1- Non-adiabatic observables





Results

2- Multicolor Photometry

Influence of the variations of the local effective temperature

Non-adiabatic computation

$$\delta m_\lambda = -\frac{2.5}{\ln 10}\,\varepsilon P_l^m(\cos i)\, b_{l\lambda}$$

$$\left[-(l-1)(l+2)\cos(\sigma t) + \left(\frac{\partial \ln F_\lambda^+}{\partial \ln T_{\rm eff}} + \frac{\partial \ln b_{l\lambda}}{\partial \ln T_{\rm eff}}\right)\left|\frac{\delta T_{\rm eff}}{T_{\rm eff}}\right| \cos(\sigma t + \psi_T) - \left(\frac{\partial \ln F_\lambda^+}{\partial \ln g} + \frac{\partial \ln b_{l\lambda}}{\partial \ln g}\right)\left|\frac{\delta g_e}{g_e}\right| \cos(\sigma t) \right]$$

Surface distortion

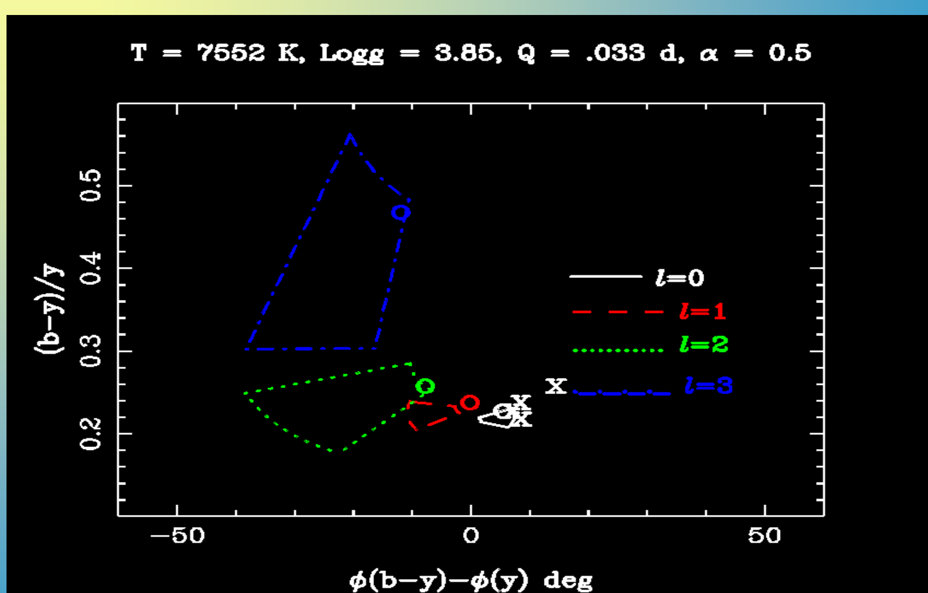
Equilibrium atmospheric models (Kurucz 1993)

$$\frac{\delta g_e}{g_e} = \frac{\delta(\partial \Phi/\partial r) - \sigma^2\,\delta r}{g}$$

Influence of the variation of the local effective gravity

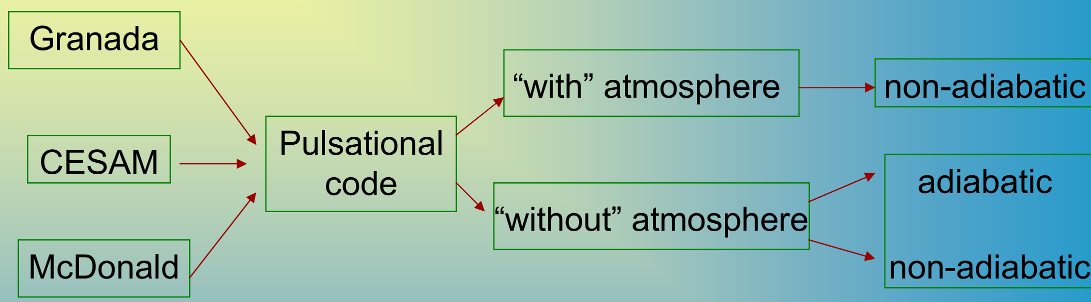
Results

2- Multicolor photometry



Summary

1. Present a code solving first order differential equations of the adiabatic and non-adiabatic stellar pulsations, including or not the atmosphere-pulsation interaction:



Plus first order perturbative rotation

Summary

For this Task 2 is important to remark:

1. The adiabatic eigenfunctions are:

a) Radial displacement ξ_r

b) Horizontal displacement ξ_h, related with the eulerian perturbation of the pressure and the gravitational potential

c) Eulerian perturbation of the gravitaitonal potential Φ’

d) Derivative of Φ’

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Summary

2. The constants are those prescribed in Task 1.

3. The code do not re-mesh the grid

4. The stellar radius is regarded as the one given for the photosphere by the equilibrium model

Conclusiones

- Fotometría multicolor → Identificación modal; Astrosismología no adiabática; Muy útil en futuras misiones espaciales

- Perspectivas futuras → Mejorar los modelos de atmósfera; Interacción rotación - pulsación; Interacción convección - pulsación

Perturbative equations in the interior

Adiabatic equations

Perturbative equations in the interior and the atmosphere

1) Stellar interior:

Inner boundary conditions (r→0):

Imposed:

1. Mass conservation
2. Kinetic moment conservation
3. Poisson equation
4. Energy conservation
5. Diffusion approximation for the radiative flux

$$\frac{d\phi}{dr} = \frac{l\phi}{r}$$

$$f(r,\theta,\varphi,t) = f_r(r) Y_l^m(\theta,\varphi) e^{i\sigma t}$$

$$\delta S = 0$$

Lower

With the spherical symmetry approximation for the star