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UNIVERSIDADE DO PORTO

Centro de Astrofísica da Universidade do Porto

Porto Oscillation Code - POSC -

Mário João P. F. G. Monteiro

Centro de Astrofísica da Universidade do Porto
and
Faculdade de Ciências da Universidade do Porto

Objectives

The code has been developed to be applied to the Sun. It determines the **frequencies of linear adiabatic radial and non-radial oscillations** for models of solar-type stars.

As input it uses a model provided in the **AMDL** format defined by Jørgen Christensen-Dalsgaard and corresponding to,

$$a_1 = \frac{m_{r,0}}{r^3}\,\frac{R^3}{M}$$

$$a_2 = -\frac{1}{\Gamma_{1,0}}\,\frac{\mathrm{d}\log P_0}{\mathrm{d}\log r} = \frac{r g_0}{c_0^2}$$

$$a_3 = \Gamma_{1,0}$$

$$a_4 = \frac{1}{\Gamma_{1,0}}\,\frac{\mathrm{d}\log P_0}{\mathrm{d}\log r} - \frac{\mathrm{d}\log \rho_0}{\mathrm{d}\log r} = \frac{r}{g_0}\,N_0^2$$

$$a_5 = \frac{4\pi r^3 \rho_0}{m_{r,0}}$$

The equations for the amplitudes

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$$\left(1-\frac{S_l^2}{\omega^2}\right)\,\frac{\tilde{P}}{\rho_0}-\frac{1}{r^2}\,\left(g_0-c_o^2\,\frac{\mathrm{d}}{\mathrm{d}r}\right)(r^2\xi_r)+\frac{S_l^2}{\omega^2}\,\tilde{\Phi}=0$$

$$\frac{1}{\rho_0}\left(\frac{g_0}{c_0^2}+\frac{\mathrm{d}}{\mathrm{d}r}\right)\,\tilde{P}-(\omega^2-N_0^2)\,\xi_r-\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r}=0$$

$$\tilde{P}+\frac{\rho_0 c_0^2 N_0^2}{g_0}\,\xi_r-\frac{S_l^2}{4\pi G}\,\tilde{\Phi}+\frac{c_0^2}{4\pi G r^2}\,\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r}\right)=0\,.$$

Equilibrium structure:

$$\rho_0\,,\,g_0\,,\,c_0^2\,,\,N_0^2$$

Radial amplitude of the eigenfunctions:

$$\xi_r\,,\,\tilde{P}\,,\,\tilde{\Phi}$$

Reduce variables

$$y_1 = \frac{\xi_r}{r}$$

$$y_2 = \frac{\omega^2}{g}\,\xi_h = \frac{1}{rg}\left(\frac{\tilde{P}}{\rho} - \tilde{\Phi}\right)$$

$$y_3 = \frac{\tilde{\Phi}}{rg}$$

$$y_4 = \frac{1}{g}\,\frac{\mathrm{d}\tilde{\Phi}}{\mathrm{d}r}$$

$$\sigma^2 = \frac{R^3}{GM}\,\omega^2$$

$$r\,\frac{\mathrm{d}y_1}{\mathrm{d}r} = (a_2-3)y_1 + \left[\frac{l(l+1)}{\sigma^2}\,a_1 - a_2\right] y_2 + a_2 y_3$$

$$r\,\frac{\mathrm{d}y_2}{\mathrm{d}r} = \left(\frac{\sigma^2}{a_1} - a_4\right) y_1 + (1+a_4-a_5)y_2 - a_4 y_3$$

$$r\,\frac{\mathrm{d}y_3}{\mathrm{d}r} = (1-a_5)y_3 + y_4$$

$$r\,\frac{\mathrm{d}y_4}{\mathrm{d}r} = a_4 a_5 y_1 + a_2 a_5 y_2 + [l(l+1) - a_2 a_5]\, y_3 - a_5 y_4$$

$$x\,\frac{\mathrm{d}\vec{y}}{\mathrm{d}x} = \mathcal{A}\cdot\vec{y}$$

Numerical scheme

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A second order scheme for the integration of the equations is used corresponding to

$$\vec{y}_{n+1} = \vec{y}_n + \frac{x_{n+1} - x_n}{2} \left[\left(\frac{\mathrm{d}\vec{y}}{\mathrm{d}x} \right)_n + \left(\frac{\mathrm{d}\vec{y}}{\mathrm{d}x} \right)_{n+1} \right]$$

This corresponds to estimate the values of the functions at a new mesh point (*n+1*) using the following expression,

$$\vec{y}_{n+1} = \left[\left(\mathcal{I} - \frac{x_{n+1} - x_n}{2\, x_{n+1}} \mathcal{A}_{n+1} \right)^{-1} \otimes \left(\mathcal{I} + \frac{x_{n+1} - x_n}{2\, x_n} \mathcal{A}_n \right) \right] \cdot \vec{y}_n$$

Resonance cavity

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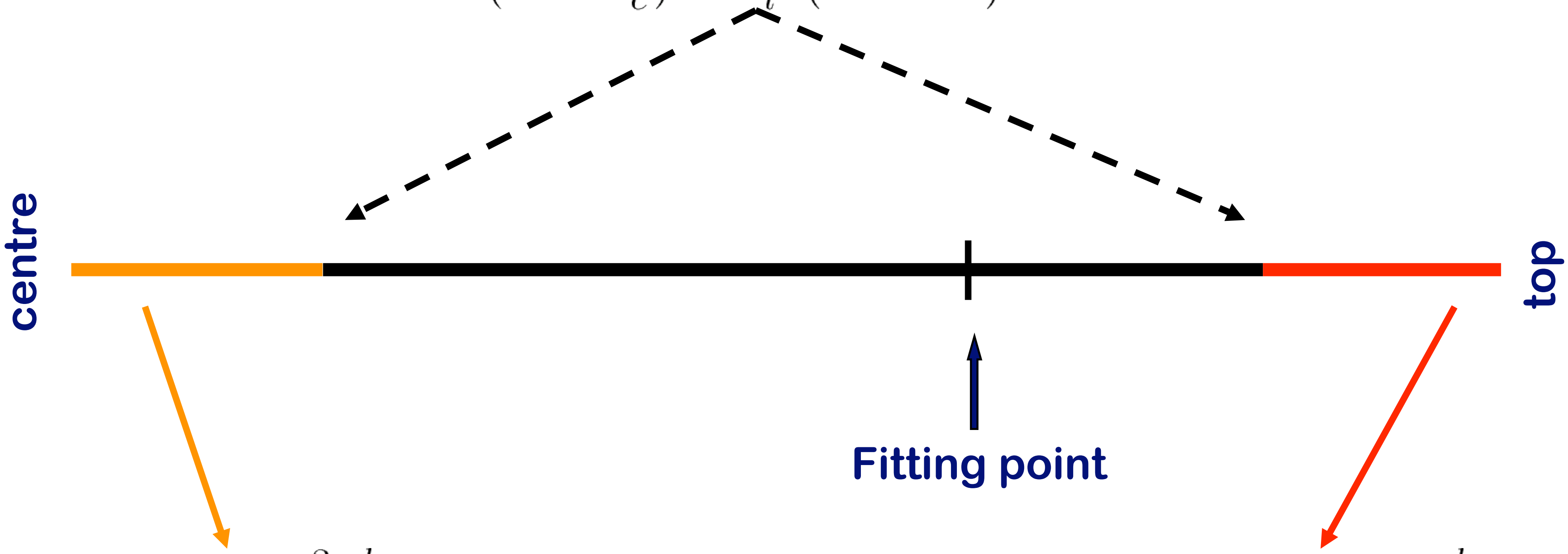
$$\omega^2\,(\omega^2-\omega_c^2) - S_l^2\,(\omega^2-N^2) = 0$$



$$\vec{y}_{\rm in} = \left(\frac{x}{x_{\rm in}}\right)^{2-l}\vec{y}$$

$$\vec{y}_{\rm out} = \left(\frac{x}{x_{\rm out}}\right)^{l}\vec{y}$$

Boundary conditions I

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At the centre we use:

$$y_{\mathrm{in},1} = \frac{4\pi\rho_c}{3}\frac{l}{\sigma^2}\, y_{\mathrm{in},2}$$

$$y_{\mathrm{in},4} = l\; y_{\mathrm{in},3}$$

An expansion for squared radius at the centre is used for the first step.

For the mesh point at the top the default boundary corresponds to an isothermal atmosphere:

$$y_{\mathrm{out},1} = 2\,\frac{\left(L^2/\sigma^2 - a_{2t}\right)\; y_{\mathrm{out},2} + a_{2t}\; y_{\mathrm{out},3}}{a_{4t} + 4 - \gamma^{1/2}}$$

$$\gamma \equiv \left(a_{2t} - a_{4t} - 4\right)^2 + 4\left(\sigma^2 - a_{4t}\right)\left(\frac{L^2}{\sigma^2} - a_{2t}\right)$$

$$y_{\mathrm{out},4} = -(l+1)\; y_{\mathrm{out},3}$$

Boundary conditions II

For an isothermal atmosphere only solutions, with a specific frequency, having a turning point at the surface (positive value of γ) are calculated.

The full reflection at the top, corresponding to a zero lagrangian pressure perturbation, is also available as;

$$y_{\mathrm{out},1} = y_{\mathrm{out},2} + y_{\mathrm{out},3}$$

The full reflection as defined by Cunha (1999) is also implemented as;

$$y_{\mathrm{out},1} = \frac{y_{\mathrm{out},2} + y_{\mathrm{out},3}}{1 + \dfrac{4+\sigma^2}{a_3 a_4}}$$

The frequencies

The frequencies are found as the solution of an eigenvalue equation obtained from the values of two independent solutions at the fitting point, as calculated with

$$y_{\mathrm{in},2} = y_{\mathrm{out},2} = 1 \qquad \& \qquad y_{\mathrm{in},3} = y_{\mathrm{out},3} = 1$$

$$y_{\mathrm{in},2} = y_{\mathrm{out},2} = 1 \qquad \& \qquad y_{\mathrm{in},3} = y_{\mathrm{out},3} = 0$$

The code searches for all solutions of the eigenvalue equation in a specified range of frequencies having the required mode degree.

The search of the zeros in frequency is optimized for *g* or *p* modes.

The mode identification is done as in Unno et al (1989) by using the normalized eigenfunctions for pressure and displacement.

Numerical precision

Richardson extrapolation is used:

$$\sigma^2 = \frac{\alpha}{\alpha - 1}\ \sigma_N^2 - \frac{1}{\alpha - 1}\ \sigma_{N'}^2 \qquad \text{with} \qquad \alpha = \left(\frac{N}{N'}\right)^2$$

Due to the low order of the integration scheme being used the precision of the frequencies is **strongly dependent on the mesh** in which the model is given.

Tipically a mesh with 6000-8000 points is used with a distribution in radius defined by the type of star and type of modes (*p* or *g*) for which the frequency is being calculated.

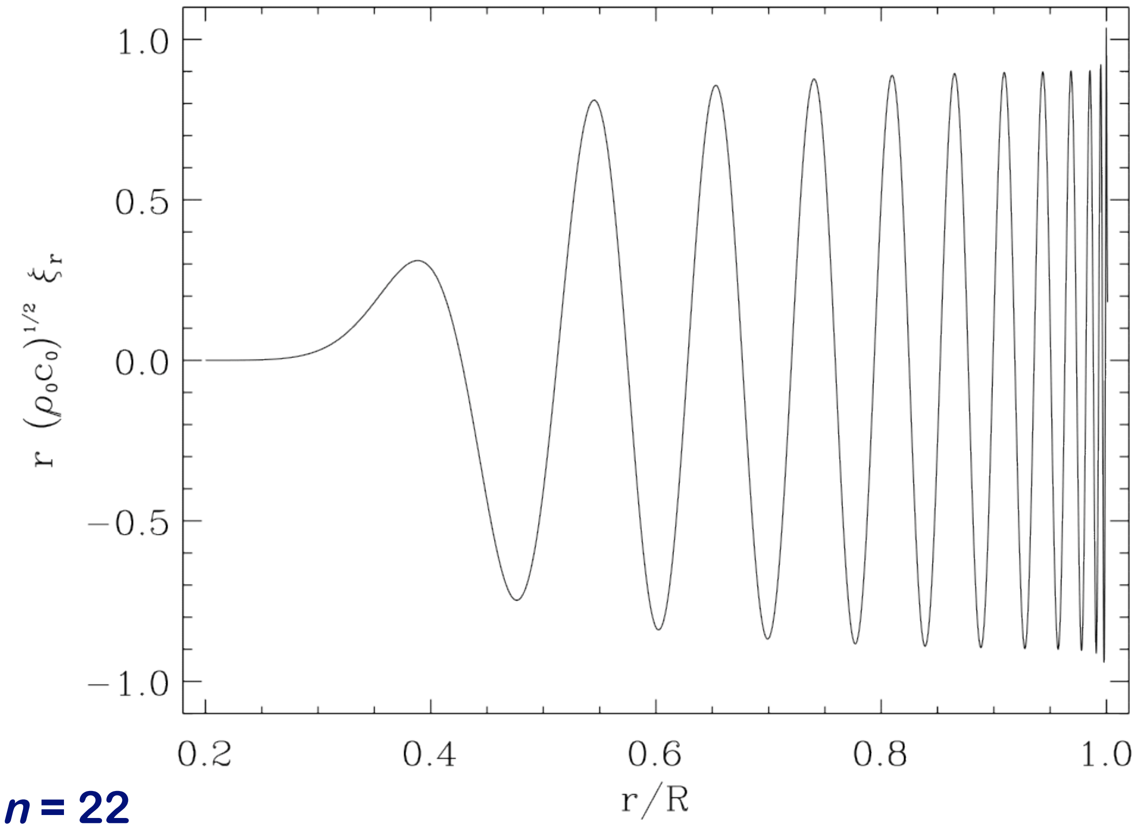
A comparison with ADIPLS using solar models gives a difference below 0.004 μHz for low and intermediate degree modes. The difference for radial modes is slightly higher (up to 0.02 μHz).

Eigenfunctions

The mode energy is calculated according to

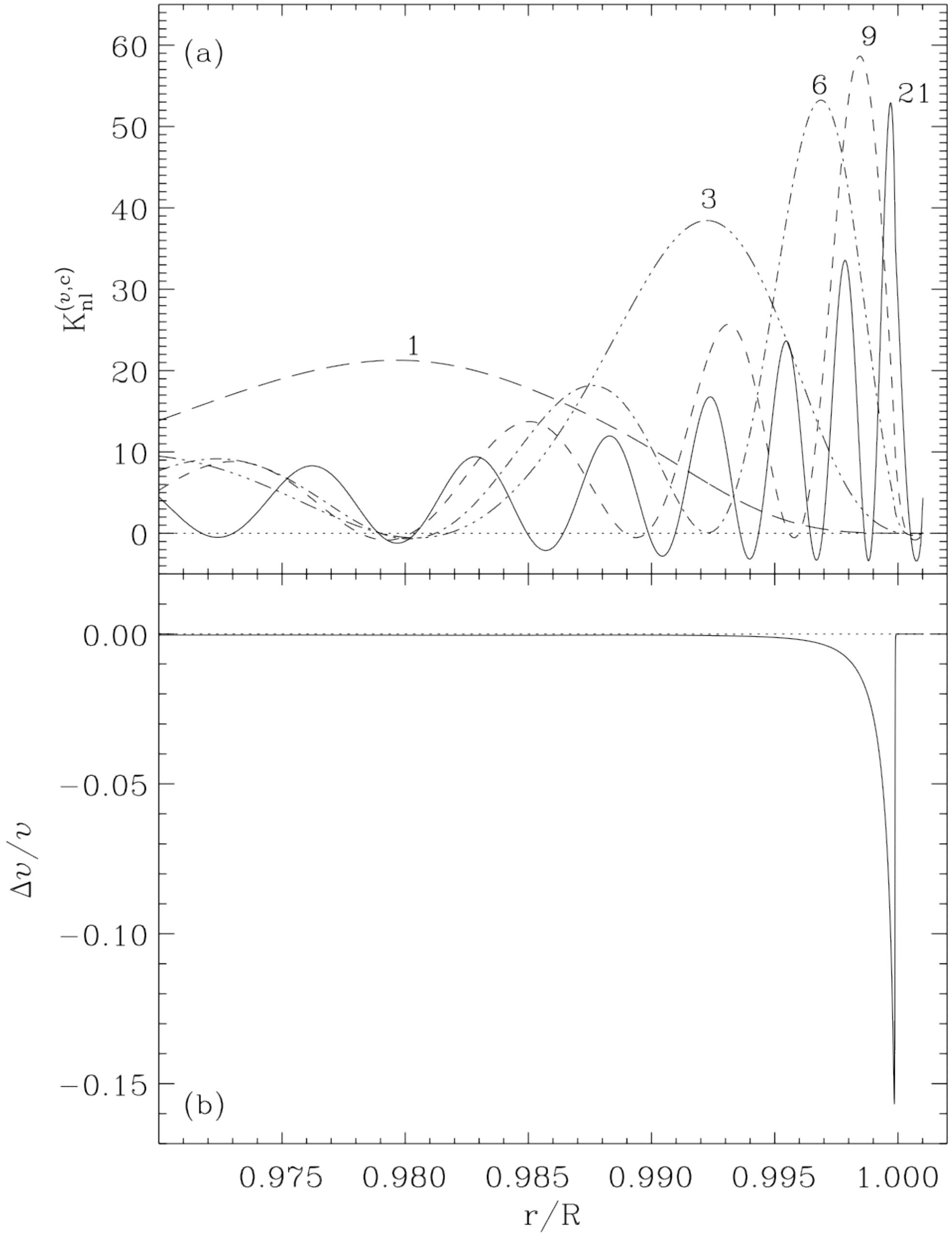
$$E_{nl} \equiv \frac{4\pi}{M} \frac{\int_0^R \left[\xi^2_{r,nl}(r) + L^2\xi^2_{h,nl}(r)\right] r^2\rho_0 \mathrm{d}r}{\xi^2_{r,nl}(R) + L^2\xi^2_{h,nl}(R)}$$

The eigenfunctions are normalized to 1 at *r* = R.

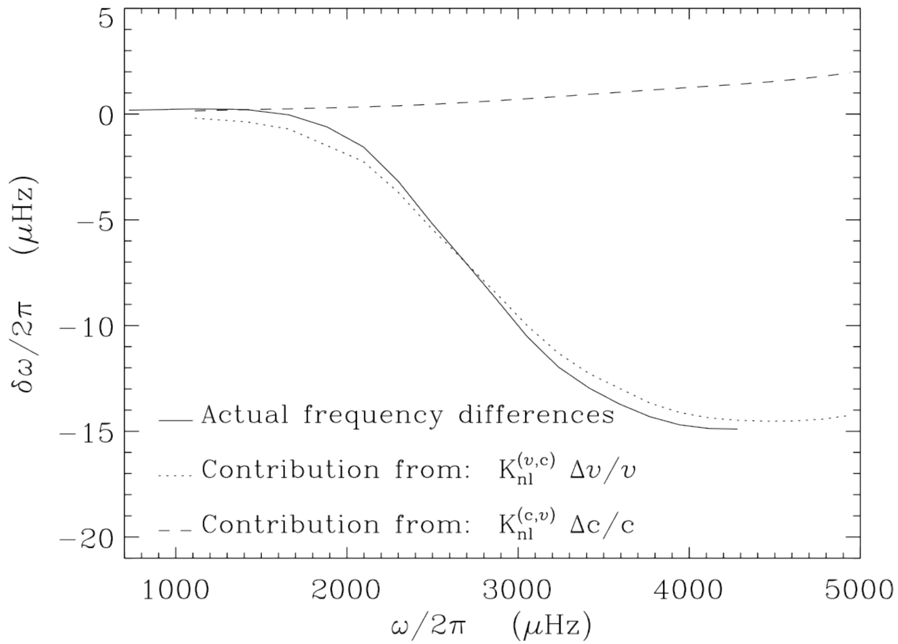


l = 20 and *n* = 22

Applications - solar convection



Kernels are also available (but have never been checked against the output from other codes).



Re-meshing

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To re-mesh the models we follow the prescription by Christensen-Dalsgaard & Berthomieu (1991);

$$f_1(x) = \frac{\left|\frac{a_2}{x^2 a_1}\right|}{\max\left|\frac{a_2}{x^2 a_1}\right|} \qquad f_2(x) = \frac{\left|\frac{a_1 a_4}{x^2}\right|}{\max\left|\frac{a_1 a_4}{x^2}\right|} \qquad f_3(x) = \frac{\left|\frac{a_2 a_3}{x^2}\right|}{\max\left|\frac{a_2 a_3}{x^2}\right|}$$

$$z(x) = \int_0^x \left[1 + C_1\ f_1(x') + C_2\ f_2(x') + C_3\ f_3(x')\right]^{1/2} \mathrm{d}x'$$

New mesh points are added when the separation in *z* of the original mesh is larger than *z*(*1*) / *N*, where *N* is the required number of mesh points. All original points are kept in the new mesh.

Parameters	MS/LM/P	MS/LM/G	PMS/IM/P
C_1	10	0.025	1000
C_2	0.01	0.1	10
C_3	0.015	0.0001	100