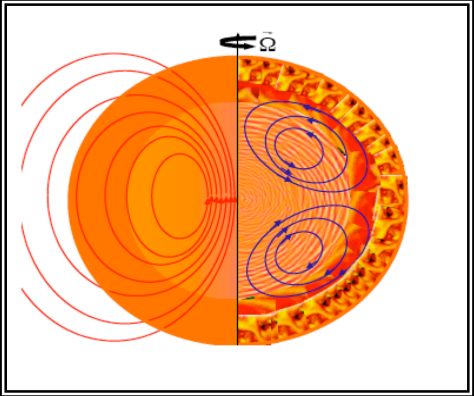
Dynamical processes in stellar radiation zones

Advances in secular magnetohydrodynamics of rotating stars



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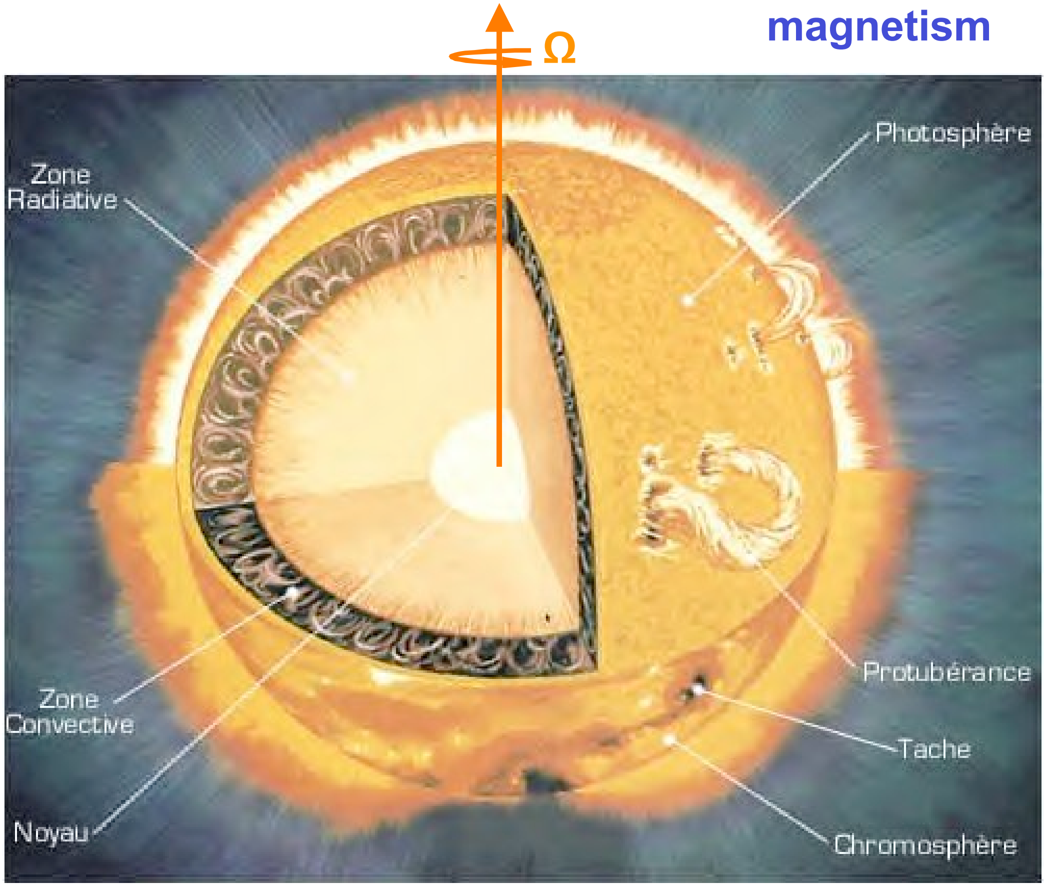
Centro de Astrofísica da Universidade do Porto, Portugal

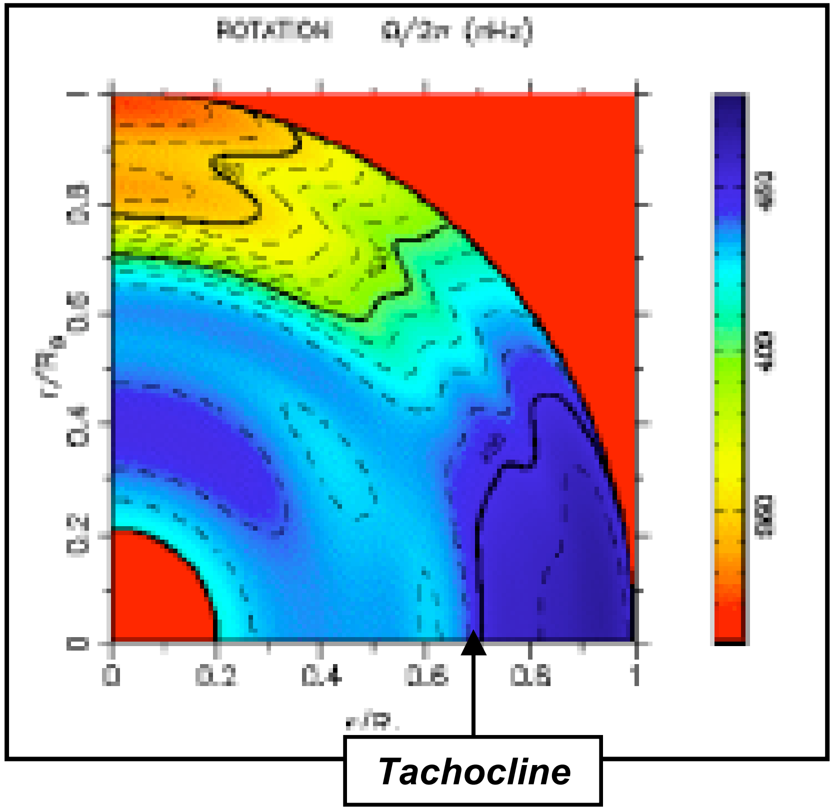
Secular magnetohydrodynamics of stellar radiation zones

Example of our Sun:

Complex magnetism



Rotation profile not understood



A coherent picture of the stars and their evolution → need to understand the dynamical transport processes which operate in their radiation zones

Major impact of differential rotation

Rotation →

- large scale **circulations**
- **turbulence**
- **magnetic activity**
- **waves**
- **tidal potential** (binary systems)
- **stellar winds** (mass loss)

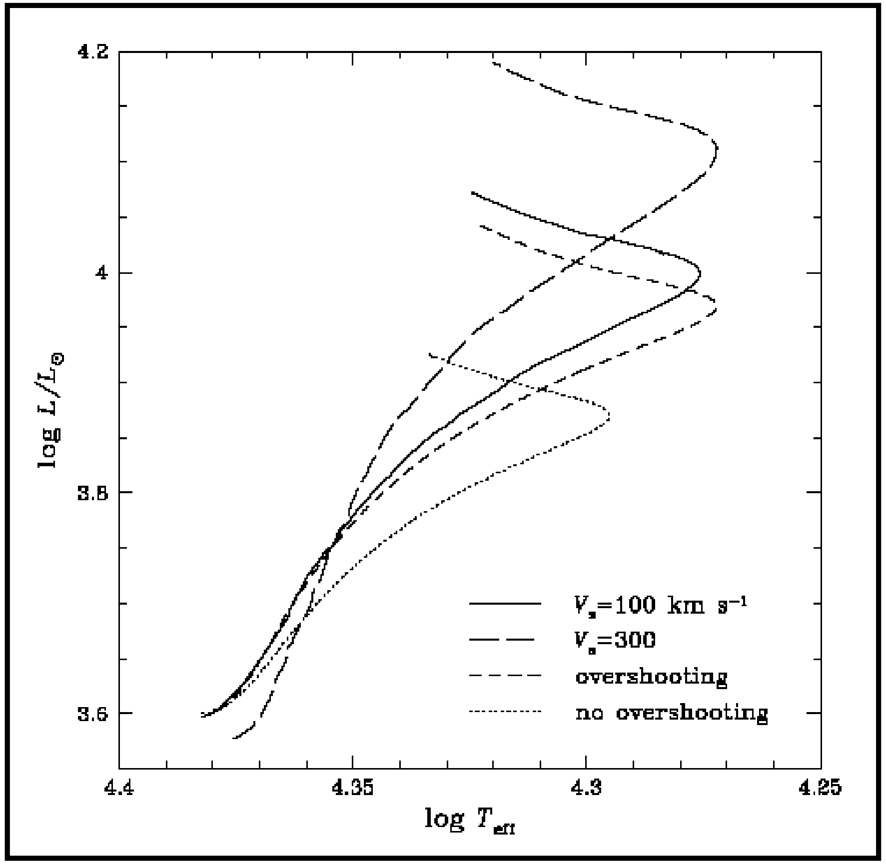
Those processes transport

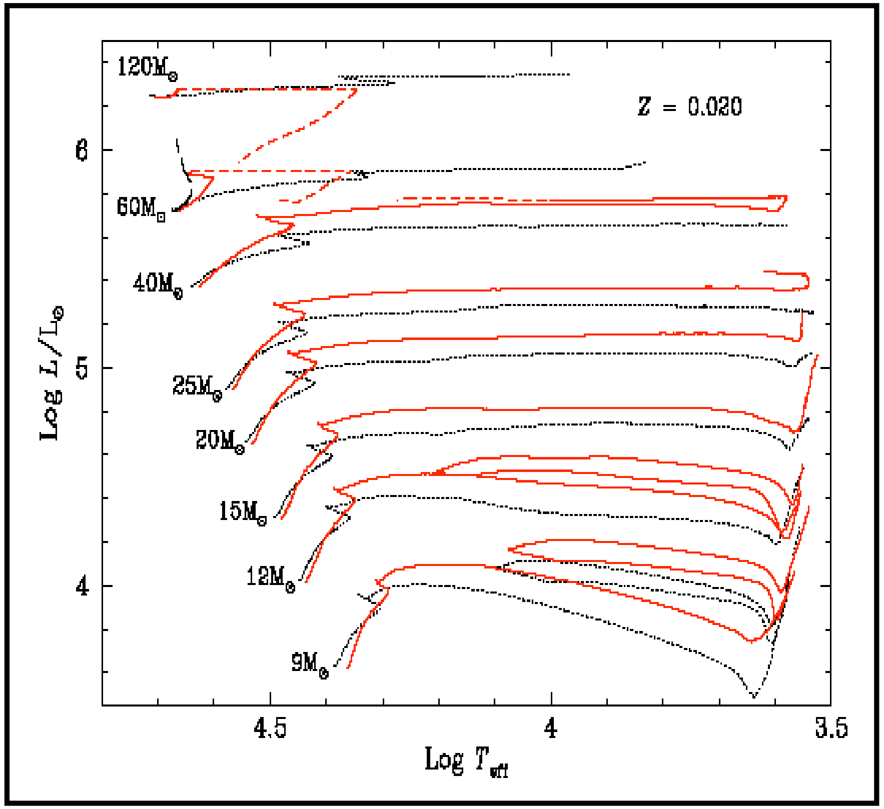
- Angular Momentum → $\Omega(r,\theta)$
- Matter → modify chemical composition & nucleosynthesis

→ **Major impact on the internal dynamics,**

the evolution and the environment of the stars

Impact on stellar evolution



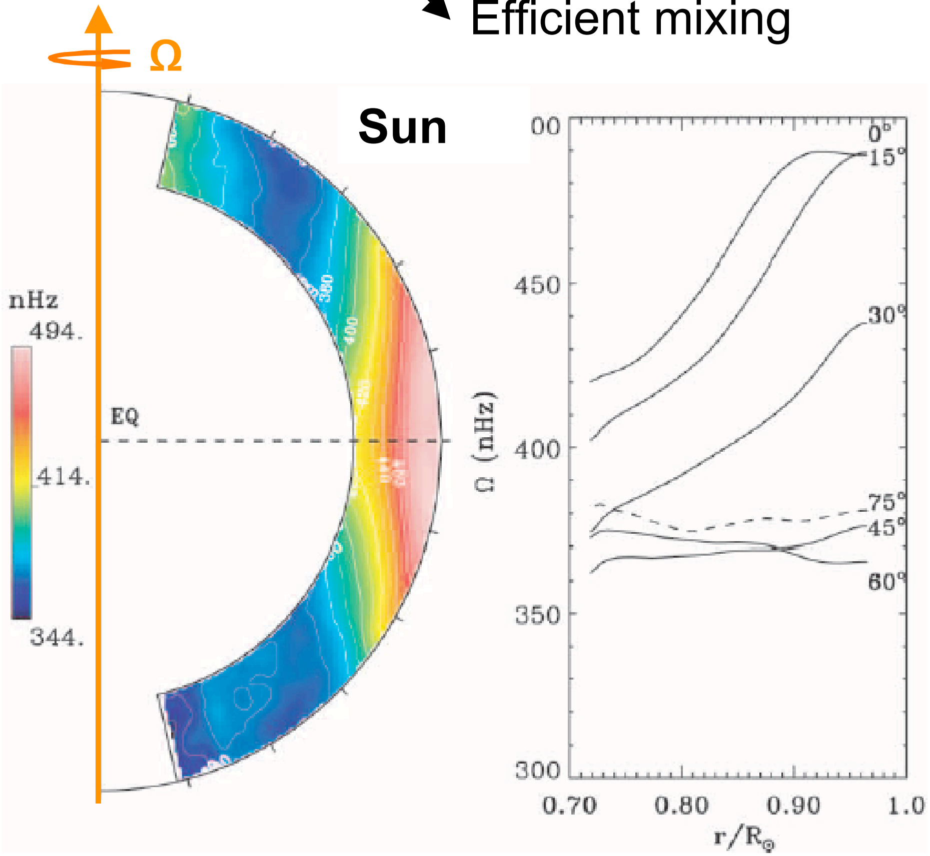


Influence of the rotation on the evolutionary track of a 9M$_\odot$ star in the HR diagram
Talon et al., 1997

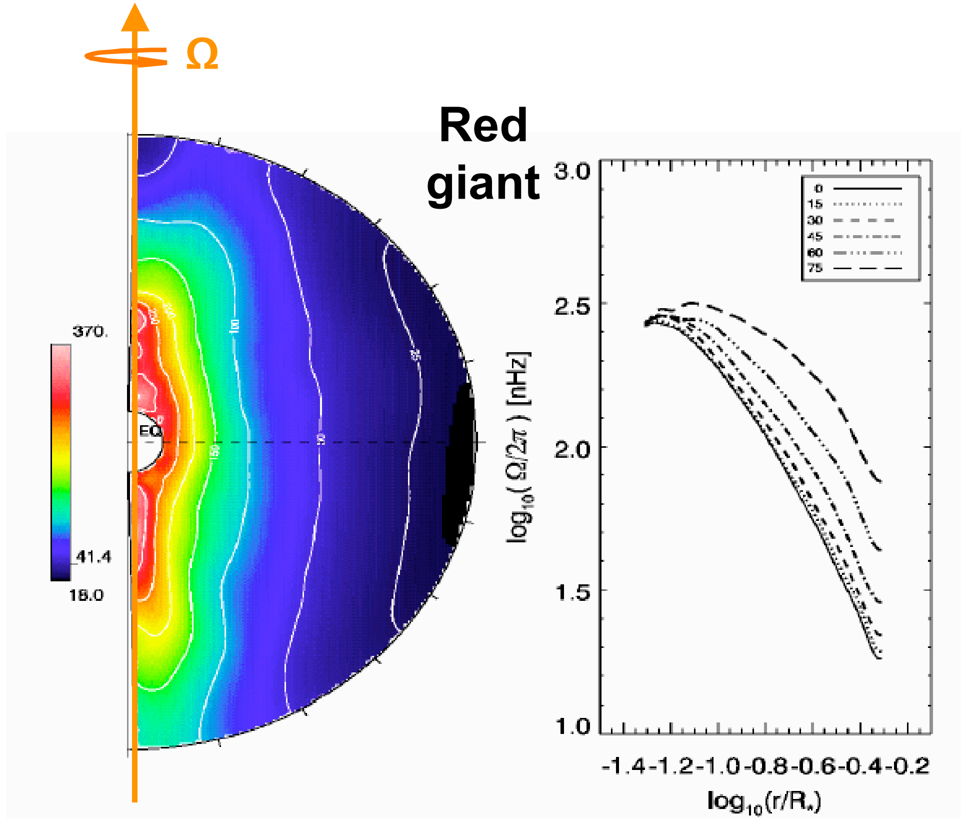
Evolutionary tracks for stars without (in black) and with (in red) rotation
Meynet & Maeder, 2000

Transport in Convection Zones

Turbulence
- → Angular Momentum transport → differential rotation
- → Efficient mixing



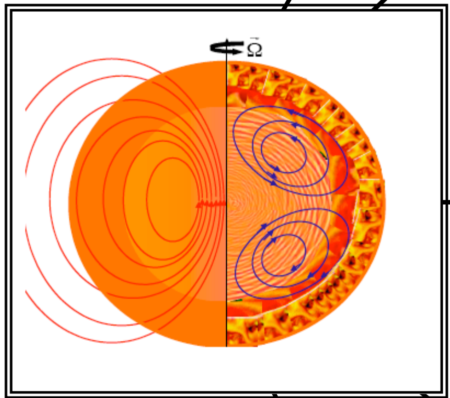
Brun & Toomre 2002



Brun & Palacios, in prep.

Massive parallel simulations - no simple prescription (yet)

Transport processes in radiation zones



Meridional circulation (diff. rot. and A. M. transport) (ADVECTION)
(Busse 1981; Zahn 1992; Maeder & Zahn 1998; Garaud 2002; Rieutord 2004; Mathis & Zahn 2004)

Turbulence (shear of the diff. rot.) (DIFFUSION)
(Talon & Zahn 1997; Lignères et al. 1999; Garaud 2001; Maeder 2003; Mathis et al. 2004)

Magnetic field

- **Secular torque**
  (Charbonneau & Mac Gregor 1993; Garaud 2002; Mathis & Zahn 2005; Brun & Zahn 2006)
- **Instabilities**
  (Maeder & Meynet 2004; Braithwaite & Spruit 2005; Brun & Zahn 2006; Zahn, Mathis, Brun in prep.)

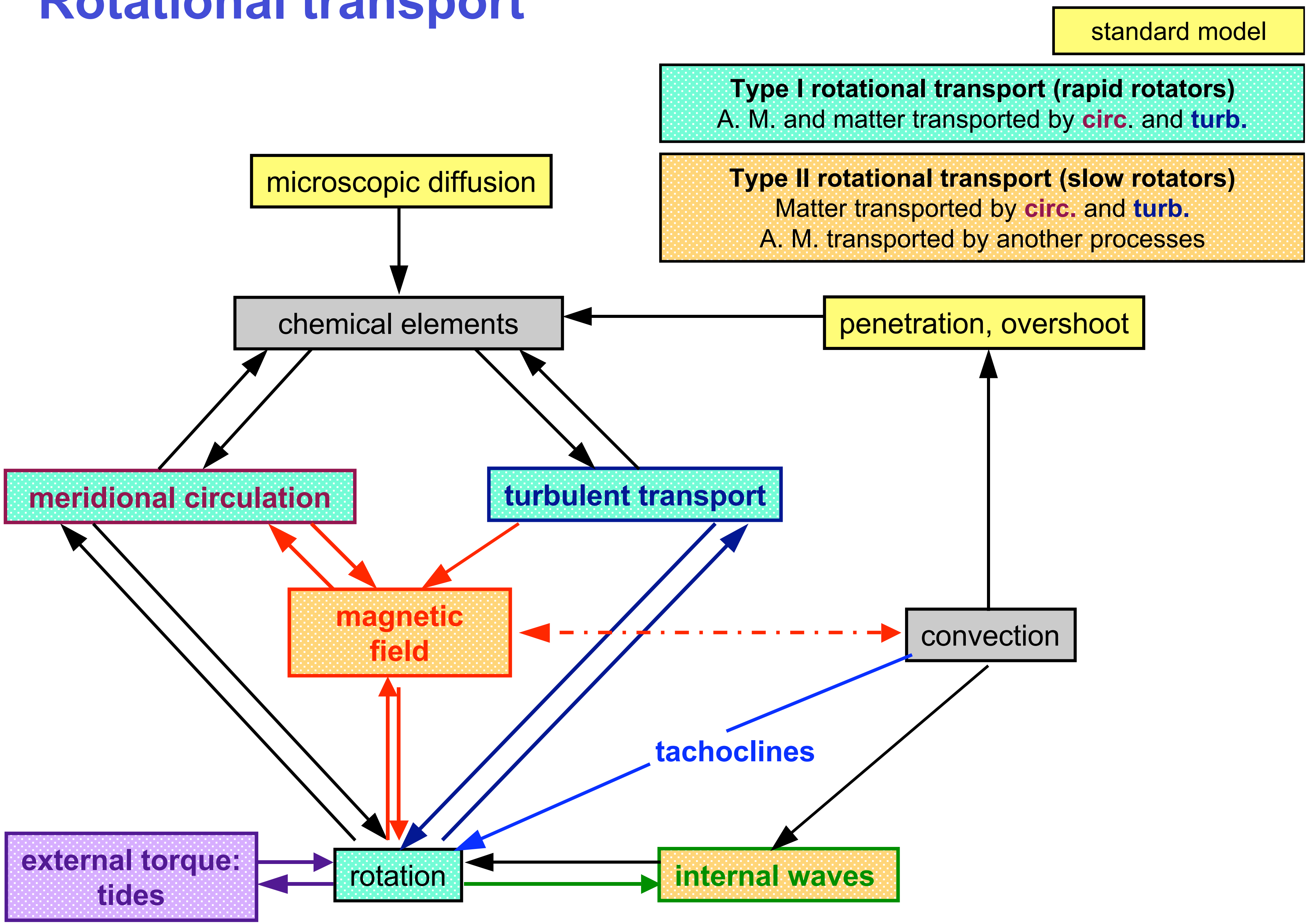
Internal waves
(Talon et al. 2002; Talon & Charbonnel 2003-2004-2005; Rogers et al. 2005-2006; Mathis & Zahn in prep.)

- excited at the borders with C. Z.
- propagating inside R. Z.
- A. M. settled where they are damped *(Goldreich & Nicholson 1989)*

Tides

- **Equilibrium tide torque** *(Zahn 1966; Mathis & Zahn in prep.)*
- **Dynamical tide** *(Zahn 1975; Mathis & Zahn)*

Rotational transport



Transport equations in stellar interiors

- Dynamics equation (Navier-Stockes equation)

$$\rho\,[\partial_t V + \underline{(V \cdot \nabla)\,V}] = -\nabla P - \rho\nabla\phi + \underline{\nabla \cdot \|\tau\|} + \left[\frac{1}{\mu_0}(\nabla \wedge B)\right] \wedge B$$

Advection

Diffusion

- Equation of continuity

$$\partial_t \rho + \nabla \cdot (\rho V) = 0$$

- Induction equation for magnetic field

$$\partial_t B - \underline{\nabla \wedge (V \wedge B)} = \underline{-\nabla \wedge (\|\eta\| \otimes \nabla \wedge B)}$$

- Equation for the transport of heat

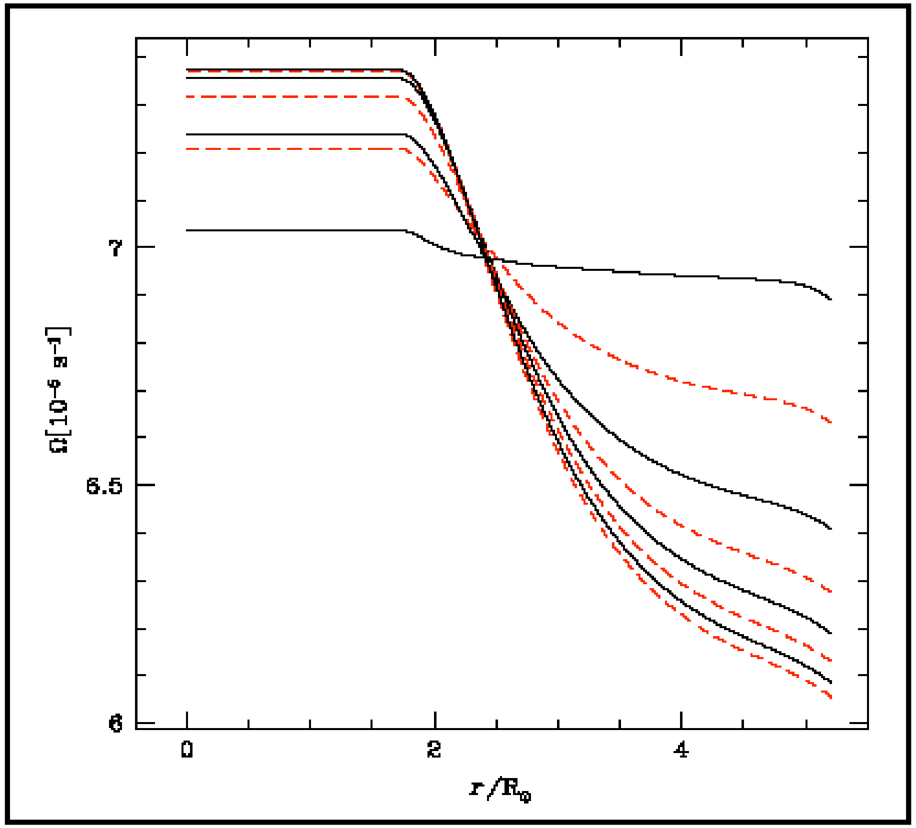
$$\rho T\,[\partial_t S + \underline{V \cdot \nabla S}] = \underline{\nabla \cdot (\chi \nabla T)} + \rho\epsilon - \nabla \cdot F + \mathcal{J}(B)$$

(+Poisson equation and the transport equation for chemicals)

Hydrostatic and thermal equilibrium hypothesis

→ equations for standard stellar evolution codes

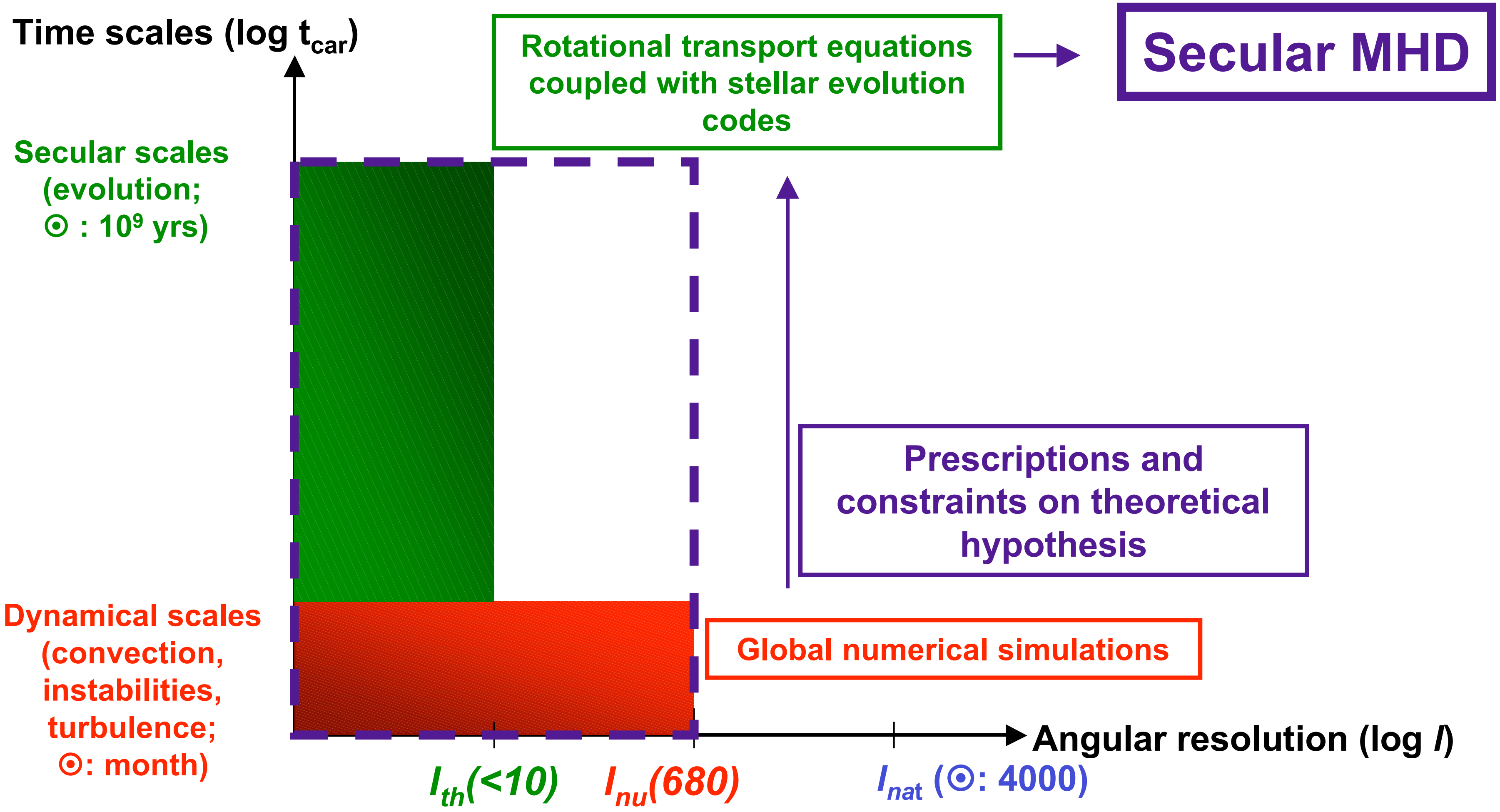
The angular momentum transport: an advective process



Evolution of the angular velocity of a 20 $M_{\odot}$ star taking a flat profile as inital condition
Meynet & Maeder, 2000

→ **Advection could not be treated as a diffusion**

A multi-scales problem in time and space



Modelling of rotational transport

Scalars (rotation, temperature, chemical concentration)

Stable stratification of radiation zones

→ Anisotropic transport ($\nu_v \ll \nu_h$, $D_v \ll D_h$)

→ allows horizontal expansion in few spherical harmonics

$$X(r,\theta,\varphi,t) = \overline{X}(r,t) + \sum_{l>0} \sum_{m=-l}^{l} \widetilde{X}_m^l(r,t)\, Y_l^m(\theta,\varphi) \quad \text{with} \quad \overline{X}(r,t) \gg \widetilde{X}_m^l(r,t)$$

Average **Fluctuation**

Vector fields

Vector fields are expanded in vectorial spherical harmonics (Rieutord 1987):

$$\boldsymbol{u}(r,\theta,\varphi,t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} u_m^l(r,t)\, \boldsymbol{R}_l^m(\theta,\varphi) + v_m^l(r,t)\, \boldsymbol{S}_l^m(\theta,\varphi) + w_m^l(r,t)\, \boldsymbol{T}_l^m(\theta,\varphi)$$

Poloidal part **Toroidal part**

where: $\boldsymbol{R}_l^m(\theta,\varphi) = Y_l^m(\theta,\varphi)\widehat{e}_r$, $\boldsymbol{S}_l^m(\theta,\varphi) = \boldsymbol{\nabla}_S Y_l^m(\theta,\varphi)$, $\boldsymbol{T}_l^m(\theta,\varphi) = \boldsymbol{\nabla}_S \wedge \boldsymbol{R}_l^m(\theta,\varphi)$ $\quad \boldsymbol{\nabla}_S = \widehat{e}_\theta \partial_\theta + \widehat{e}_\varphi \frac{1}{\sin\theta}\partial_\varphi$

⇨ **Allows to separate variables in transport equations → modal equations in r and t only**

Preliminary definitions

- Internal macroscopic velocity field:

$$\boldsymbol{V} = r\sin\theta\,\Omega(r,\theta)\,\widehat{\mathbf{e}}_\varphi + \dot{r}\widehat{\mathbf{e}}_r + \boldsymbol{\mathcal{U}}_M(r,\theta) + \boldsymbol{u}(r,\theta,\varphi,t)$$

Differential rotation | **Contraction -dilatation** | **Meridional circulation** | **Waves velocity field**

where

$$\Omega(r,\theta) = \overline{\Omega}(r) + \widehat{\Omega}(r,\theta) = \overline{\Omega}(r) + \sum_{l>0} \Omega_l(r)\,Q_l(\theta)$$

Average | **Fluctuation**

and $\boldsymbol{\mathcal{U}}_M = \sum_{l>0}\left[U_l(r)P_l(\cos\theta)\widehat{e}_r + V_l(r)\frac{\mathrm{d}P_l(\cos\theta)}{\mathrm{d}\theta}\widehat{e}_\theta\right]$ with $V_l(r) = \frac{1}{l(l+1)\rho r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho r^2 U_l\right)$

Anelastic approximation

- Temperature and mean molecular weight:

$$T(r,\theta) = \overline{T}(r) + \delta T(r,\theta) \quad \text{where} \quad \delta T(r,\theta) = \sum_{l\geq 2}\left[\Psi_l(r)\overline{T}\right]P_l(\cos\theta)$$

$$\mu(r,\theta) = \overline{\mu}(r) + \delta\mu(r,\theta) \quad \text{where} \quad \delta\mu(r,\theta) = \sum_{l\geq 2}\left[\Lambda_l(r)\overline{\mu}\right]P_l(\cos\theta)$$

- Magnetic field:

$$\boldsymbol{B}(r,\theta) = \boldsymbol{\nabla}\wedge\boldsymbol{\nabla}\wedge\left(\xi_P(r,\theta)\widehat{e}_r\right) + \boldsymbol{\nabla}\wedge\left(\xi_T(r,\theta)\widehat{e}_r\right)$$

$$\begin{cases} \xi_P(r,\theta) = \sum_{l=1}^{\infty}\xi_0^l(r)\,Y_l^0(\theta) \\ \xi_T(r,\theta) = \sum_{l=1}^{\infty}\chi_0^l(r)\,Y_l^0(\theta) \end{cases}$$

Poloidal part | **Toroidal**

Transport equations system

A. M.:

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\overline{\Omega}\right)-\frac{1}{5r^2}\partial_r\left(\rho r^4\overline{\Omega}U_2\right)=\frac{1}{r^2}\partial_r\left(\rho\nu_v r^4\partial_r\overline{\Omega}\right)+\overline{\Gamma}_{\mathcal{F}_{\mathcal{L}}}(\boldsymbol{B})-\frac{1}{r^2}\partial_r\left[r^2\mathcal{F}_J(r)\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}=\partial_t+\dot{r}\partial_r$$

idem $\Omega(\theta)$

Advection | Diffusion | Lorentz torque | Waves | Lagrangian derivative

Induction equation:

$$\begin{cases}\frac{\mathrm{d}}{\mathrm{d}t}\xi_0^l-r\mathcal{P}_{\mathrm{Ad};l}(\Omega,U_l,\boldsymbol{B})=\eta_h r\Delta_l\left(\frac{\xi_0^l}{r}\right)\\ \frac{\mathrm{d}}{\mathrm{d}t}\chi_0^l+\partial_r(\dot{r})\chi_0^l-\mathcal{T}_{\mathrm{Ad};l}(\Omega,U_l,\boldsymbol{B})=\left[\partial_r\left(\eta_h\partial_r\chi_0^l\right)-\eta_v l(l+1)\frac{\chi_0^l}{r^2}\right]\end{cases}$$

Heat:

Non stationarity

$$\frac{M(r)}{L(r)}C_p\overline{T}\frac{\mathrm{d}\Psi_l}{\mathrm{d}t}+\frac{M(r)}{L(r)}C_p\overline{T}\left[\Phi\frac{\mathrm{d}\ln\overline{\mu}}{\mathrm{d}t}\Lambda_l+\frac{U_l(r)}{H_p}(\nabla_{ad}-\nabla)\right]=\mathcal{T}_l(r)+\frac{M(r)}{L(r)}\frac{\mathcal{J}_l(\boldsymbol{B})}{\overline{\rho}}$$

$$\begin{aligned}\mathcal{T}_l=\;&2\left[1-\frac{\overline{f}_{\mathcal{P}}(\Omega,\boldsymbol{B})}{4\pi G\overline{\rho}}-\frac{\left(\overline{\epsilon}+\overline{\epsilon}_{\mathrm{grav}}\right)}{\epsilon_m}\right]\frac{\widetilde{g}_l(\Omega,\boldsymbol{B})}{\overline{g}}+\frac{\widetilde{f}_{\mathcal{P},l}(\Omega,\boldsymbol{B})}{4\pi G\overline{\rho}}-\frac{\overline{f}_{\mathcal{P}}(\Omega,\boldsymbol{B})}{4\pi G\overline{\rho}}(-\delta\Psi_l+\varphi\Lambda_l)\\ &+\frac{\rho_m}{\overline{\rho}}\left[\frac{r}{3}\partial_r\left(H_T\partial_r\Psi_l-(1-\delta+\chi_T)\Psi_l-(\varphi+\chi_\mu)\Lambda_l\right)-\frac{l(l+1)H_T}{3r}\left(1+\frac{D_h}{K}\right)\Psi_l\right]\\ &+\frac{\left(\overline{\epsilon}+\overline{\epsilon}_{grav}\right)}{\epsilon_m}\left\{\left(H_T\partial_r\Psi_l-(1-\delta+\chi_T)\Psi_l-(\varphi+\chi_\mu)\Lambda_l\right)+(f_\epsilon\epsilon_T-f_\epsilon\delta+\delta)\Psi_l+(f_\epsilon\epsilon_\mu+f_\epsilon\varphi-\varphi)\Lambda_l\right\}\end{aligned}$$

Perturbing force

Thermal diffusion

Nuclear energy production and heating due to gravitational adjustements

Thermal wind equation (baroclinic equation):

$$\varphi\Lambda_l-\delta\Psi_l=\frac{r}{\overline{g}}\mathcal{D}_l(\Omega,\boldsymbol{B})$$

Transport with gravito-inertial waves

We consider a 'shellular' rotation:

$$\Omega(r,\theta) = \overline{\Omega}(r) = \overline{\Omega}_s + \delta\overline{\Omega}(r) \text{ with } \delta\overline{\Omega}(r) << \overline{\Omega}_s$$

Uniform rotation: waves structure

Differential rotation: thermal diffusion

and we define the characteristic frequencies:

$$\begin{cases} \sigma_s = \sigma_0 + m\overline{\Omega}_s \\ \sigma(r) = \sigma_0 + m\overline{\Omega}(r) = \sigma_s + m\delta\overline{\Omega}(r) \end{cases}$$

m>0 - retrograde (extraction)
m<0 - prograde (deposit)

Local frequency (Doppler shifted)

Inertial frame

Corotating frame

We get:

$$4\pi r^2 \mathcal{F}_J(r) = \sum \left\{ 4\pi r_c^2 \mathcal{F}_J(k,m,\sigma;r_c) \exp\left[-\tau_{k,m}\left(r,\delta\overline{\Omega}(r);\nu\right)\right]\right\}$$

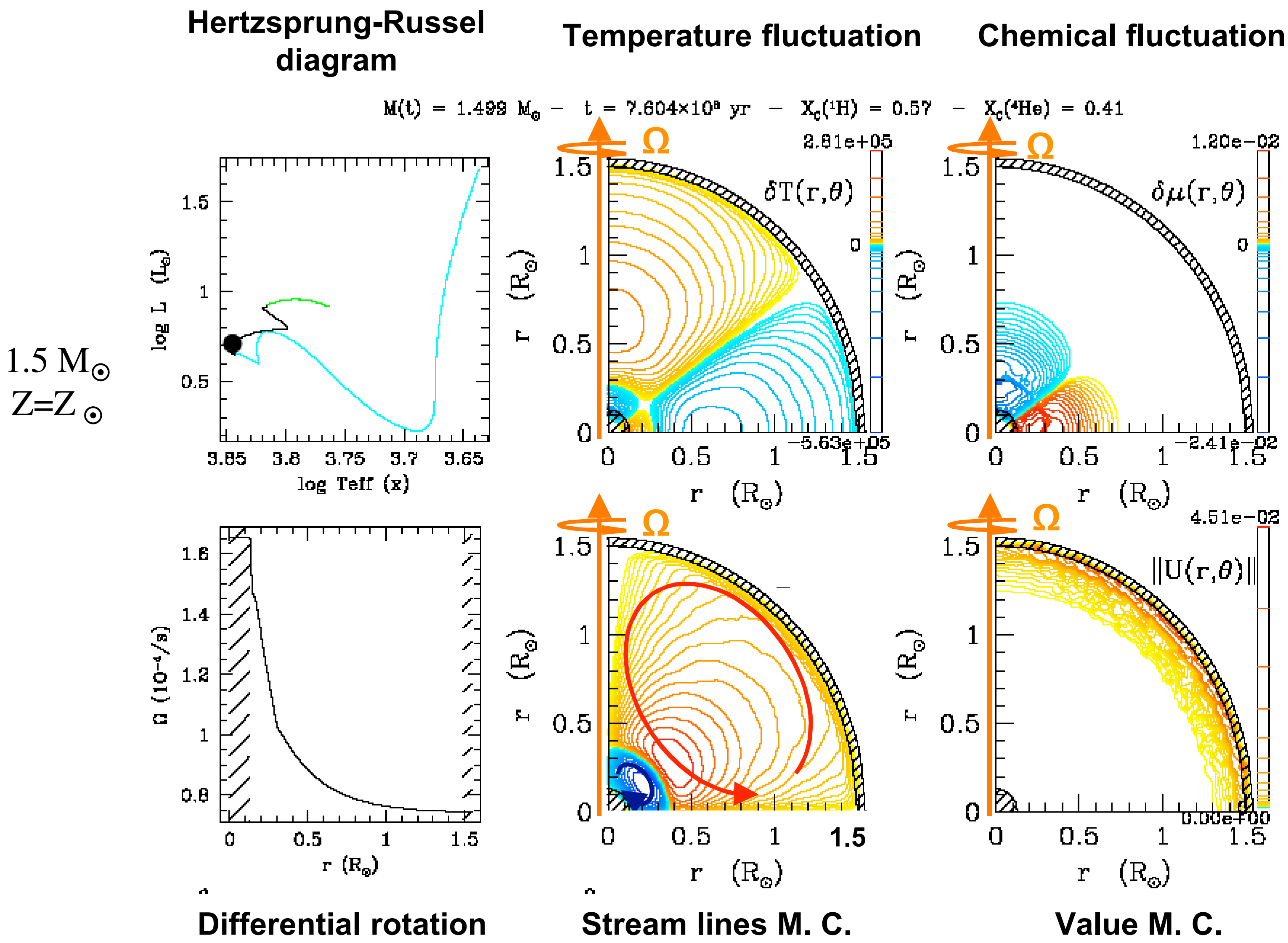
Spectrum excited by convection

flux at the base of the CZ

where $\tau_{k,m}\left(r,\delta\overline{\Omega}(r);\nu\right) = \int_r^{r_c} \frac{K k_{V;k,m}^2}{|V_{g;k,m}|} dr' = \Lambda_{k,m}^{\frac{3}{2}}(\nu) \int_r^{r_c} \left\{ K \frac{N^2}{\sigma^4(r')} \left[N^2 - \sigma^2(r')\right]^{\frac{1}{2}} \right\} \frac{dr'}{r'^3}$ and $\nu = \frac{2\overline{\Omega}_s}{\sigma_s}$

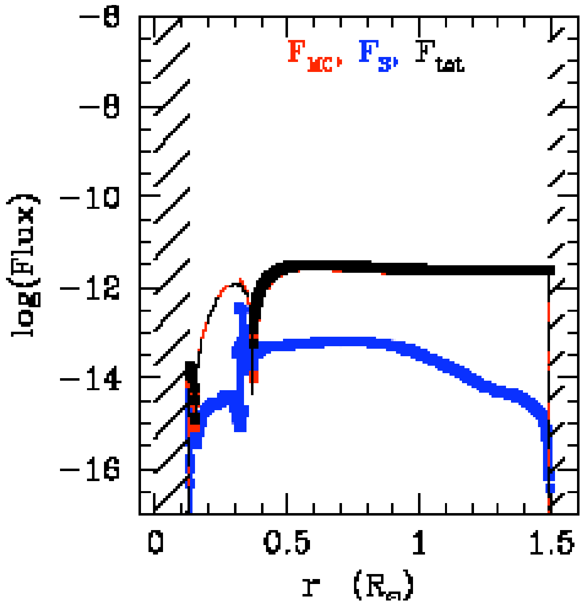
Numerical simulation of secular transport

Hydrodynamical case with Ω(r,θ)=Ω(r) (l=2); STAREVOL CODE

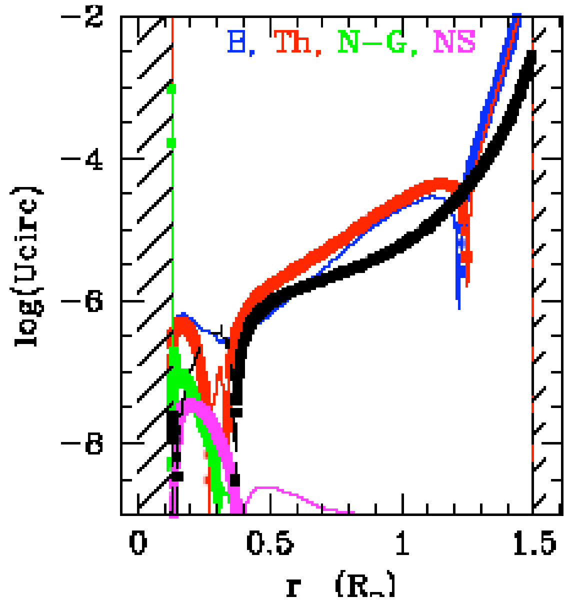


Decressin, Mathis et al.; en prep.

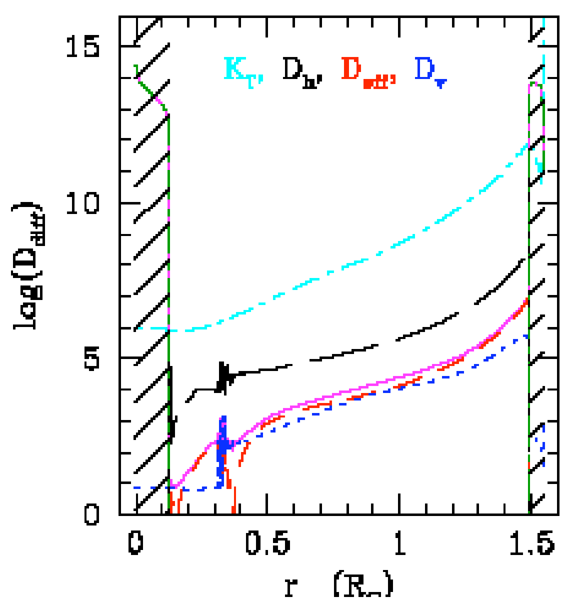
Diagnosis and identification



Flux of Angular Momentum:



Terms meridional circulation



Transport coefficients

-Meridional circulation $R_\odot^4 F_{MC}(r) = \frac{1}{5}\rho r^4 \overline{\Omega} U_2$

-Shear induced turbulence $R_\odot^4 F_S(r) = \rho \nu_v r^4 \partial_r \overline{\Omega}$

Work is in progress to implement differential rotation in latitude and transport by magnetic field and gravito-inertial waves (crucial to explain the internal rotation profile of the Sun) and the associated diagnosis (IGW: Eggenberger & Mathis)

⇒ Hydrodynamical (& MHD) vision of stellar evolution ready for helio and asteroseismic predictions and diagnosis

Type I Rotational Transport

The same processes (circulation and turbulence) are responsible for the transport of angular momentum and the mixing of chemicals

Successes:

-properties of massive stars (correct prediction of surface abundances and of population ratio (blue and red SG); Meynet 2004 and references therein)

-Li abundance on blue side of Li gap (Talon & Charbonnel 1998)

-peculiar abundances of subgiants (Palacios et al. 2003, Palacios et al. 2004)

Weaknesses: for late type stars, predicts

-fast rotating core
≠ helioseismology (Pinsonneault et al. 1989; Chaboyer et al. 1995; Matias & Zahn 1997)

-strong destruction destruction of ^{9}Be in Sun
≠Balachandran & Bell 1998; may be explained by tachocline mixing

-mixing correlated with loss of angular momentum
≠ Li in tidally locked binaries (Balachandran 2002)
≠ little dispersion in the Spite plateau

→ **Another process is responsible for the transport of angular momentum**

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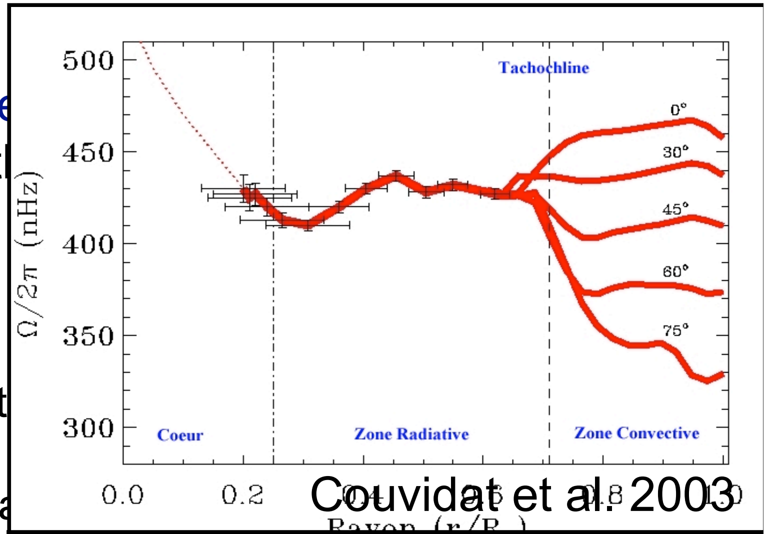
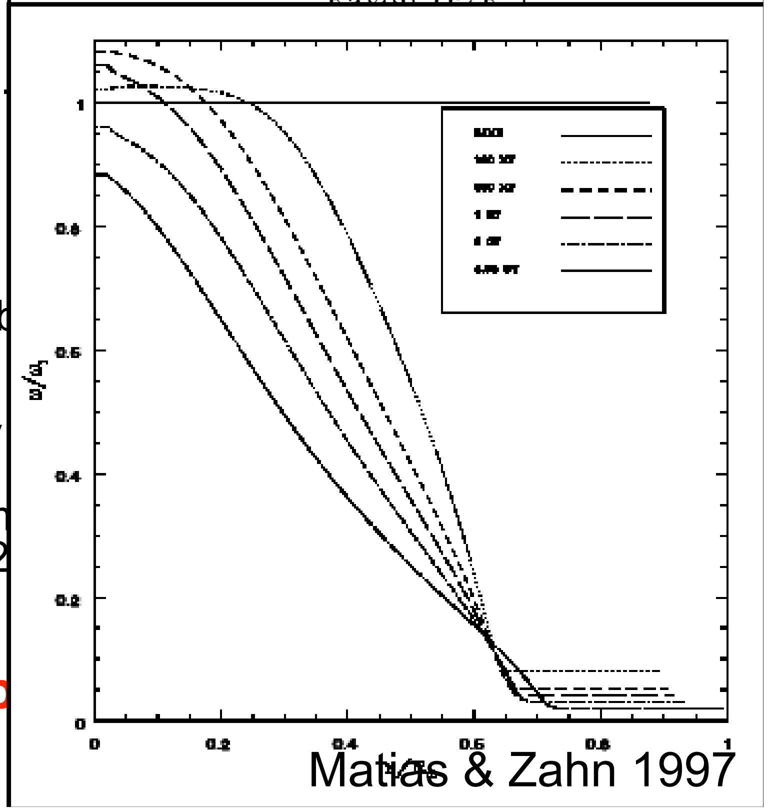
⟶ **Another process is resp**
for the transport of angular



Couvidat et al. 2003


Mathis & Zahn 1997

Type II Rotational Transport

Circulation and turbulence are responsible for
the mixing of chemicals;

Another process operates for the transport of angular momentum;
has indirect impact on mixing, by shaping the rotation profile

Magnetic field ?

Internal Gravity Waves ?

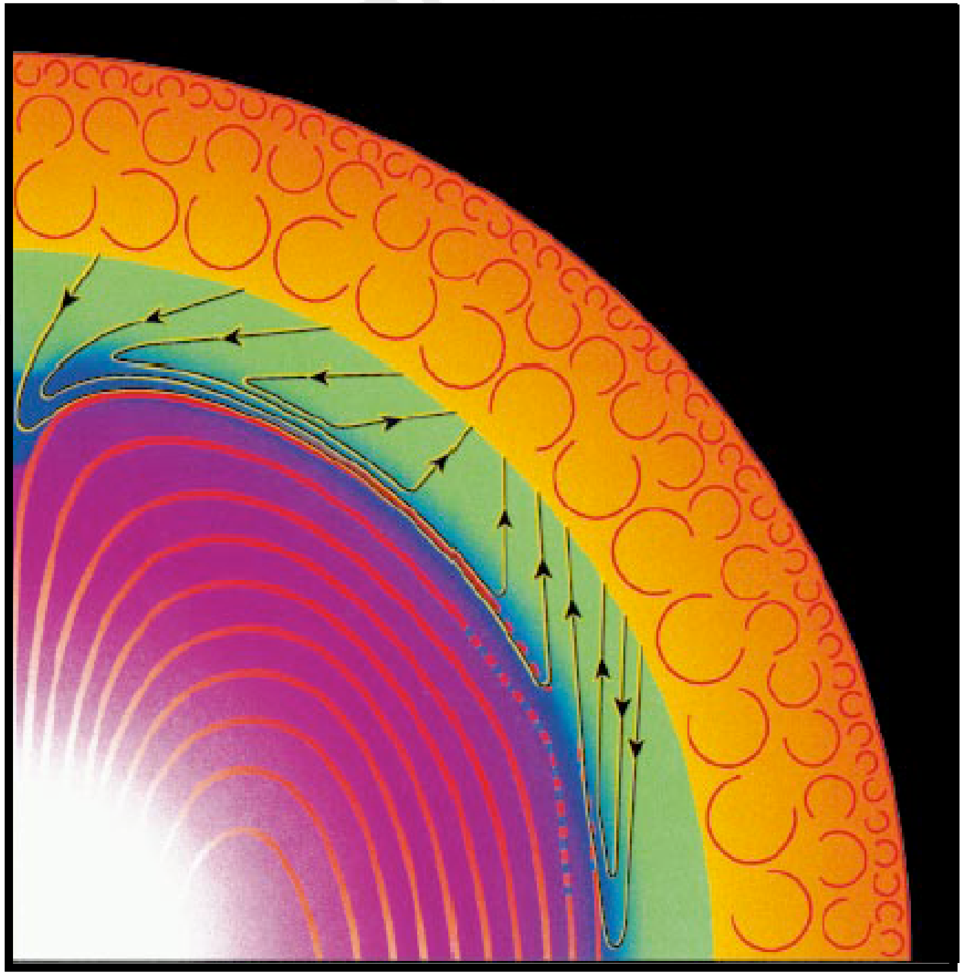
Magnetic field in radiative zones

-Does it prevent the spread of tachocline?
-Does it enforce uniform rotation?

Convection Zone
Dynamo field

Tachocline

Radiation Zone
Fossil field



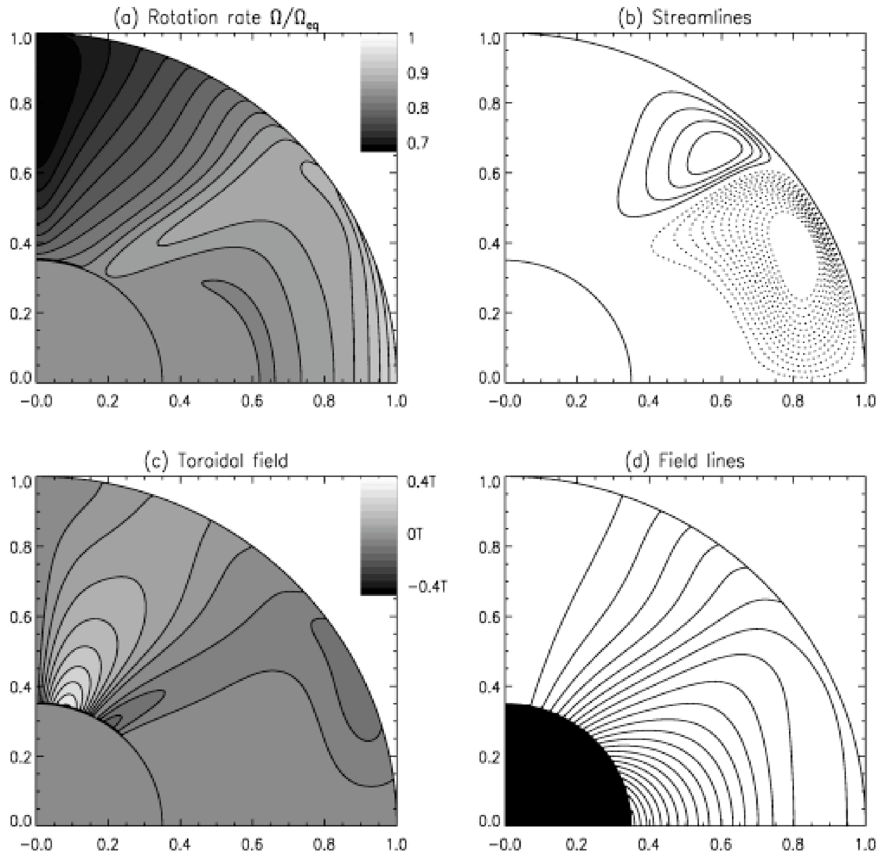
Gough & McIntyre 1998

Role of the magnetic field in radiation zones

Fossil field: 2D stationary solutions

(Garaud 2002)

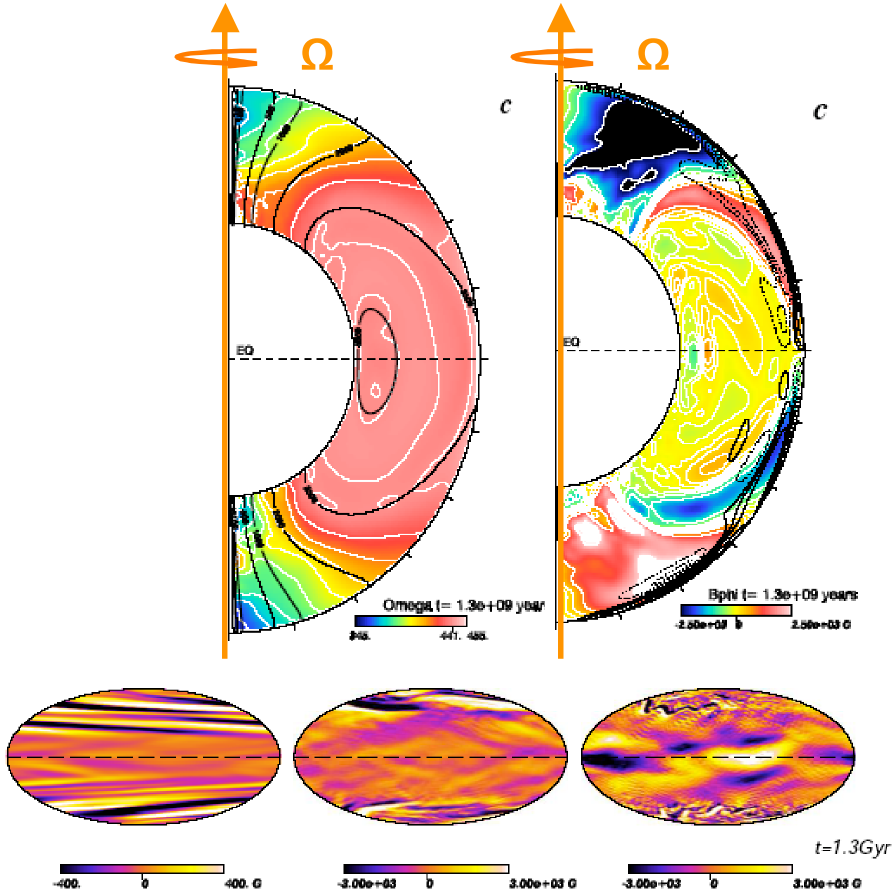
intermediate field case (13000 G)



At high latitudes poloidal field threads through CZ enforces diff. rotation

Ferraro's law

3D solutions Brun & Zahn 2006



Ferraro's law and

3D non-axisymmetric MHD instabilities

Dynamo field: rapid models of tachocline

Forgacs-Dajka 2004

Internal Gravity Waves

a crucial transport process for the evolution of solar-type stars

Study of 1.2 M$_\odot$ star

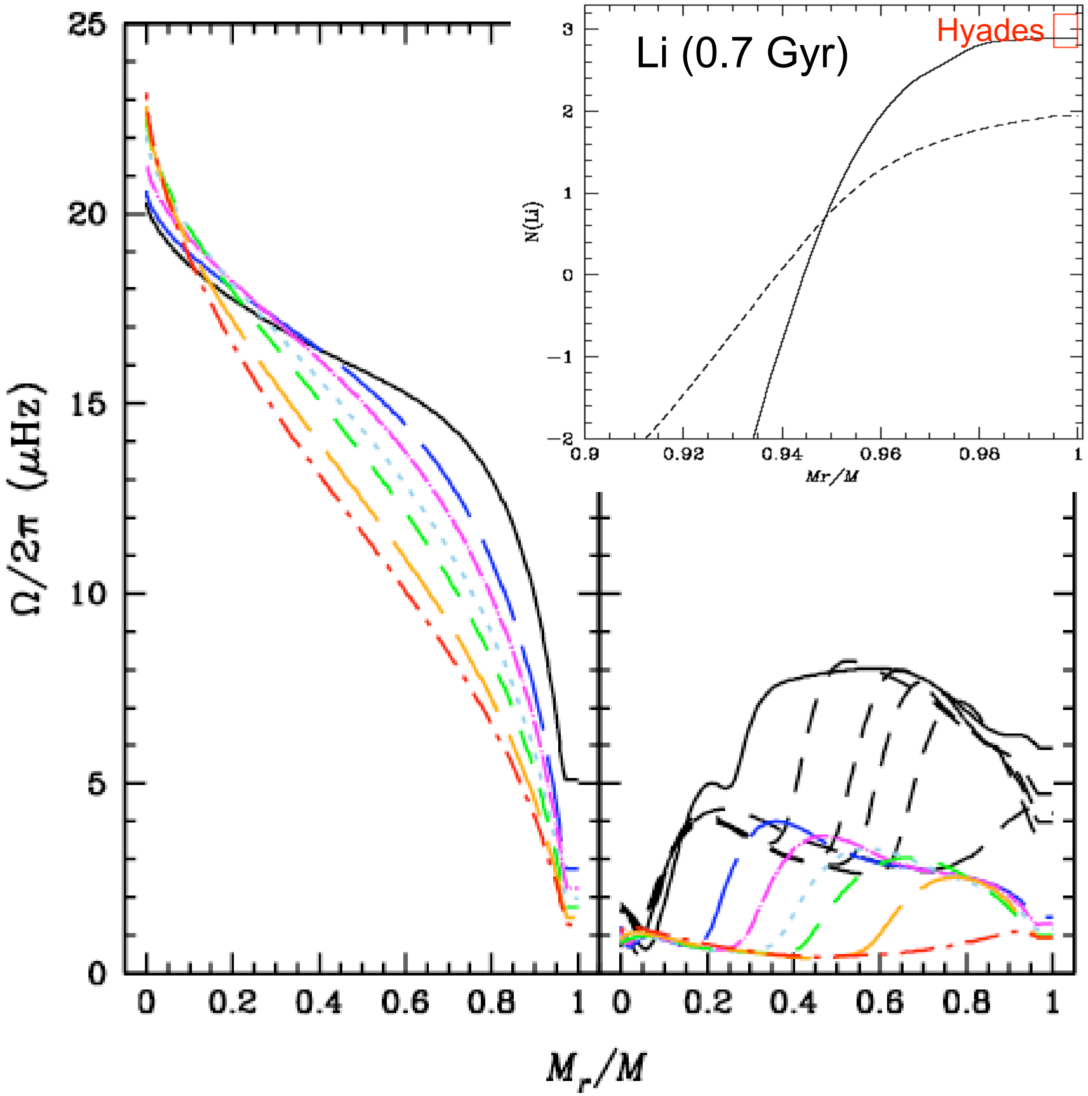
- initial angular velocity: 50 km/s & Kawaler braking law

- All the hydrodynamical processes (except magnetic field)

0.2, 0.5, 0.7, 1.0, 1.5, 3.0, 4.6 Gyrs

Type I Rot. Transp. (without IGW)

Type II Rot. Transp. (with IGW)



Charbonnel & Talon 2005,
Talon & Charbonnel 2005

Asteroseismic effects of rotational transport

Structural changes due to rotational transport ⟶ indirect effects on oscillations frequencies

Example: 1.5 $M_\odot$ star with V_i = 150 km s^{-1} (**Eggenberger, PhD Thesis**)

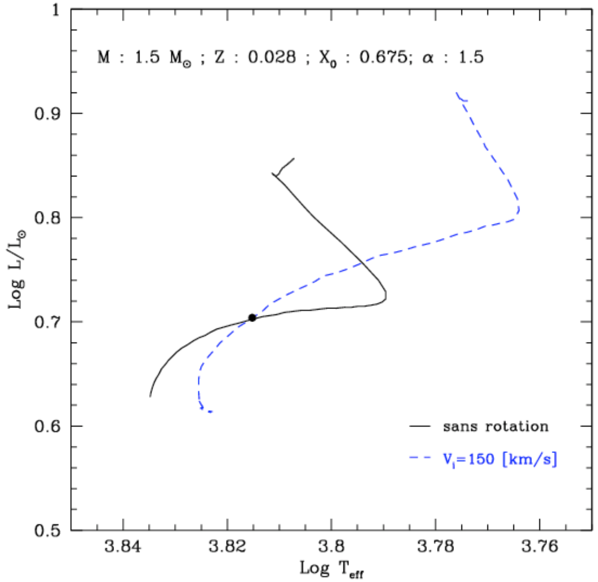
Asymptotic theory

$$\nu_{n,l} \approx \left(n + \frac{l}{2} + \varepsilon\right)\Delta\nu - \frac{l\,(l+1)}{6}\delta\nu_{0,2}$$

$$\Delta\nu = \nu_{n,l} - \nu_{n-1,l} = \left(2\int_0^R \frac{\mathrm{d}r}{c_s}\right)^{-1}$$

$$\delta\nu_{l,l+2} = \nu_{n,l} - \nu_{n-1,l+2} \approx -(4l+6)\frac{\Delta\nu}{4\pi^2\nu_{n,l}}\int_0^R \frac{\mathrm{d}c_s}{\mathrm{d}r}\frac{\mathrm{d}r}{r}$$

	Without rotation	V_i = 150 km s^{-1}
$\Delta\nu$	70.40 μHz	69.94 μHz
$\delta\nu_{02}$	5.07 μHz	5.76 μHz
X_c	0.330	0.443



Asteroseismic effect due to a dynamical processes: the case of the horizontal turbulence

Eggenberger, PhD Thesis

Physics included in models:

-Geneva evolution code (Meynet & Maeder 2000, A&A, 361, 101)

-microscopic diffusion (Richard et al. 1996, A&A, 312, 1000)

-magnetic braking (Kawaler 1988, ApJ, 333, 236)

Asteroseismic analysis:

-Aarhus oscillation code (Christensen-Dalsgaard 1997)

-modes $\ell \leq 3$

Initial parameters:

-1 $M_\odot$ and solar calibration of Y_i, $(Z/X)_i$, α_{MLT} et K

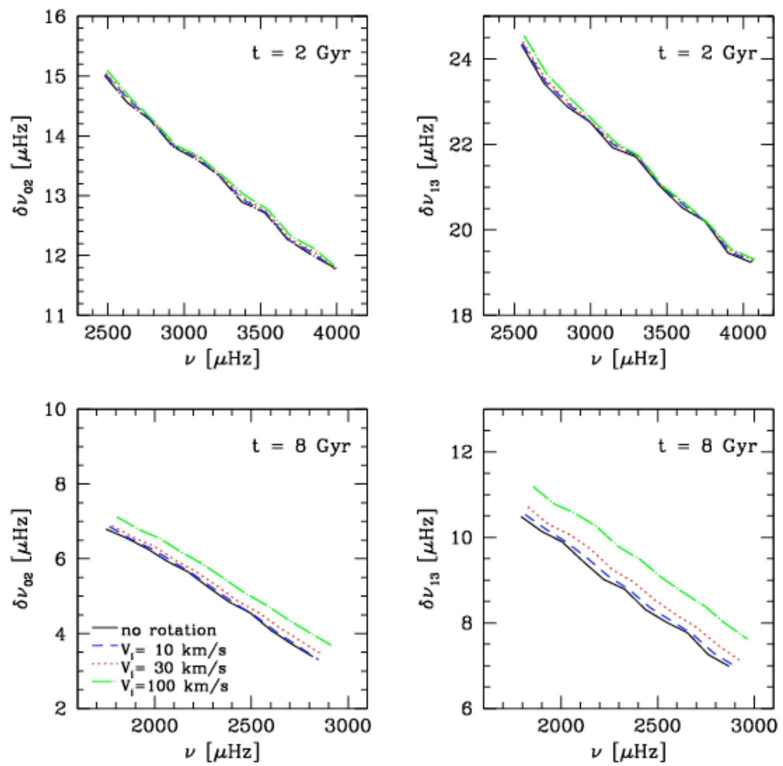
-V_i = 0, 10, 30 et 100 km s^{-1}

-3 prescriptions for the horizontal turbulence:

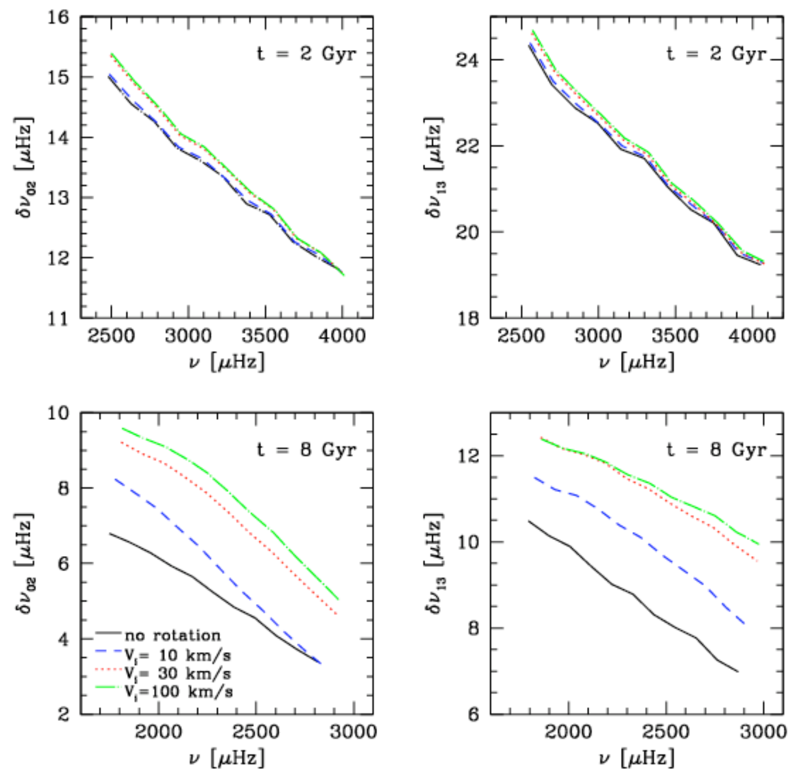
D_h Zahn (1992), Maeder (2003) et Mathis et al. (2004)

Asteroseismic effect due to a dynamical processes: the case of the horizontal turbulence

Eggenberger, PhD Thesis



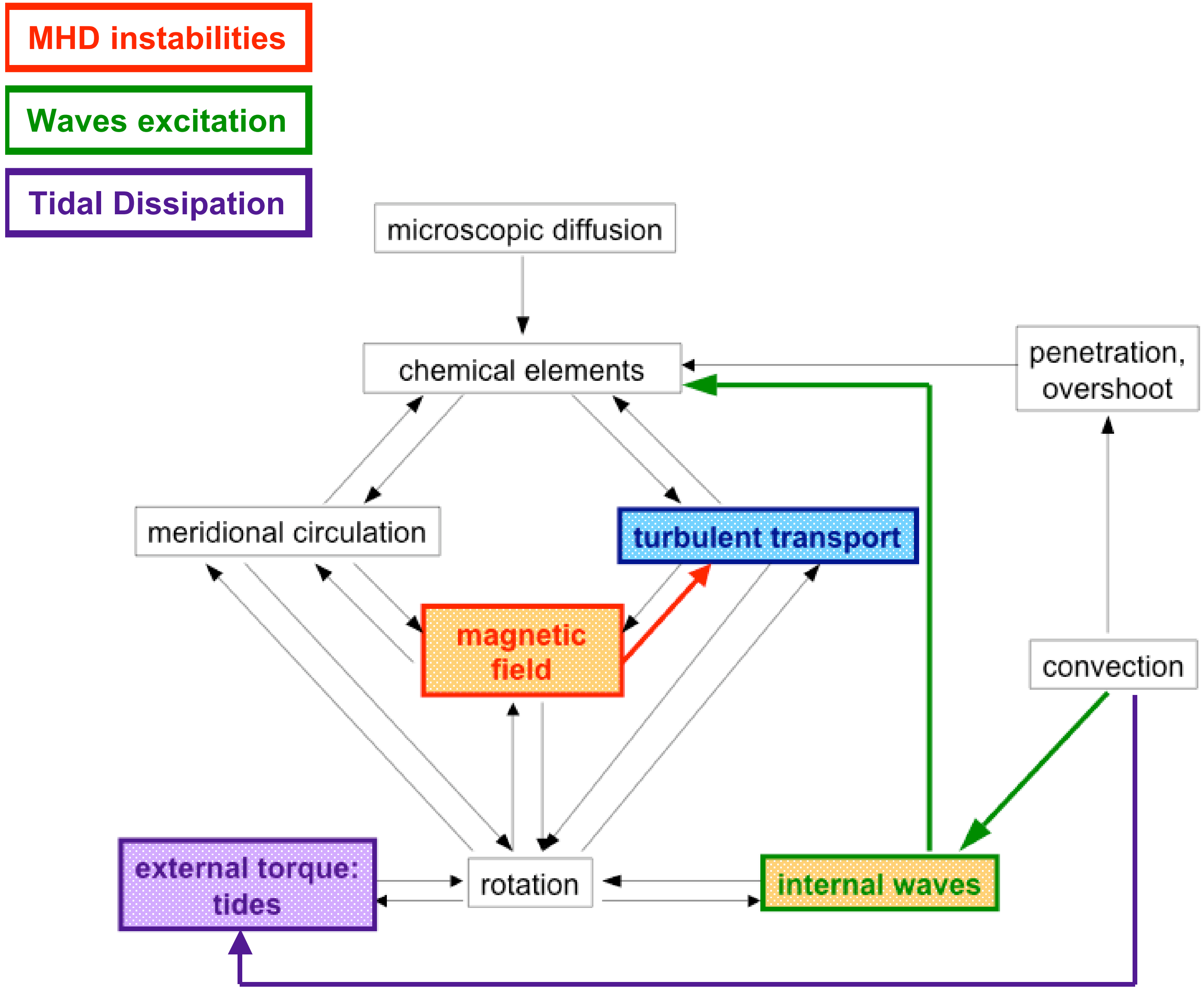
D_h Zahn 1992



D_h Maeder 2003

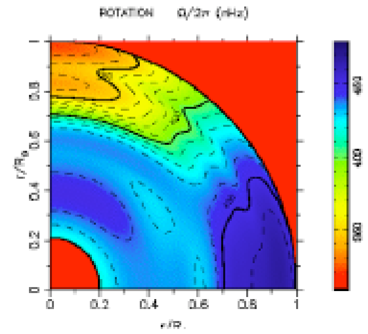
→ New prescriptions for the horizontal transport increase transport and mixing and thus the rotational effects on frequencies

What should be done

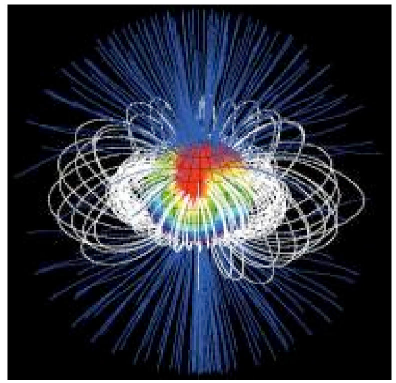


Major impact on:

-Stellar rotation



-Stellar magnetism

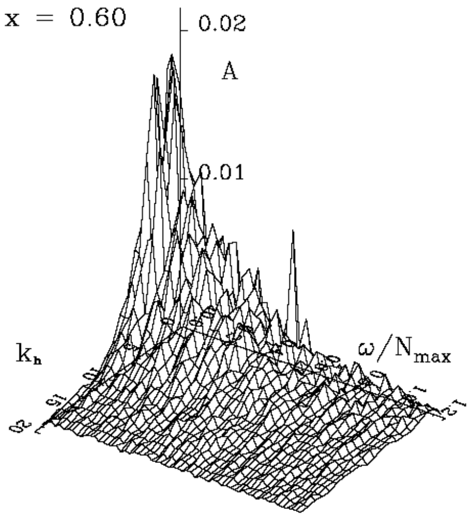


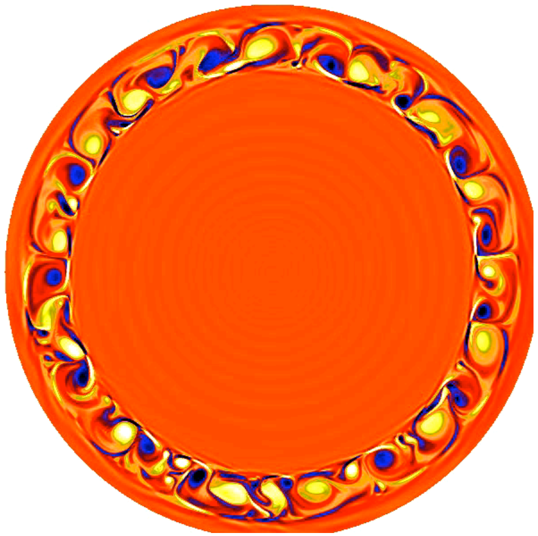
-Stellar evolution

Excitation of gravity (gravito-inertial) waves

Numerical simulations of penetrative convection

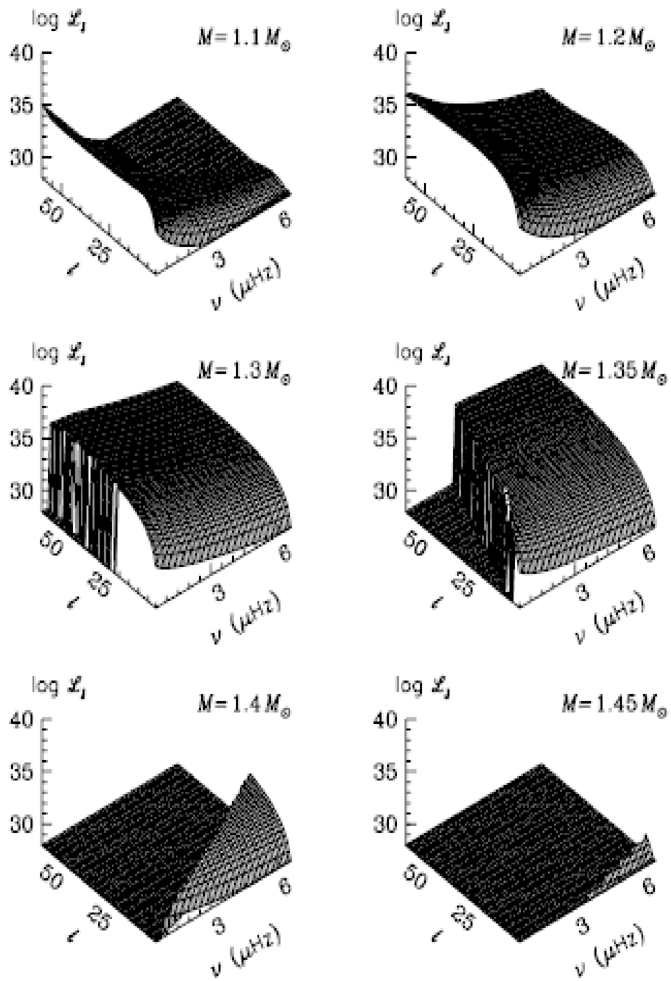
Spectrum and flux of the excited waves
Kiraga et al. 2003-2005, Dintrans 2005,
Rogers & Glatzmaier 2005-2006





Analytical treatment

Goldreich, Murray & Kumar 1994 used
by Talon & Charbonnel 2003-2004-2005



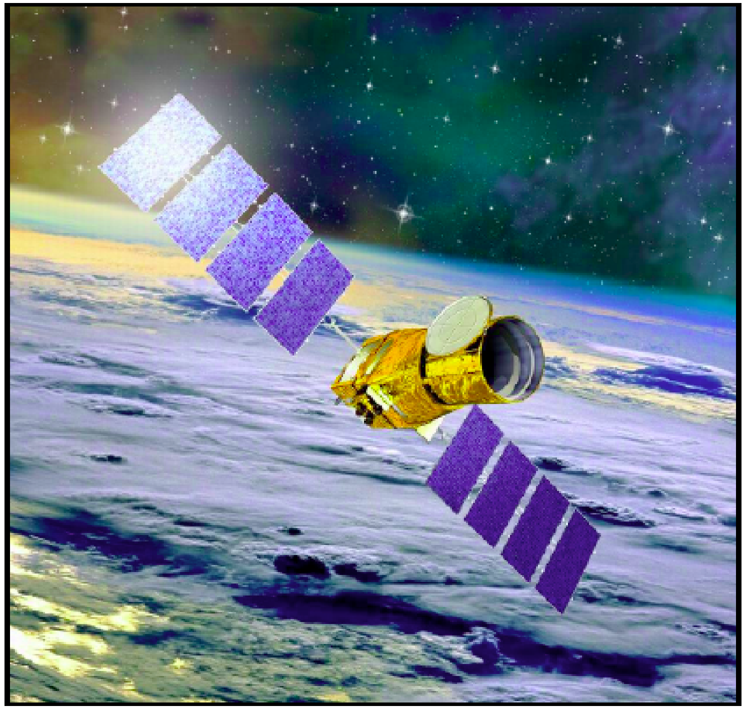
Numerical and semi-analytical treatment of the excitation of gravito-inertial waves

(A.-S. Brun, S. Mathis, J.-P. Zahn;
S. Mathis, K. Belkacem, R. Samadi, M.-J. Goupil)

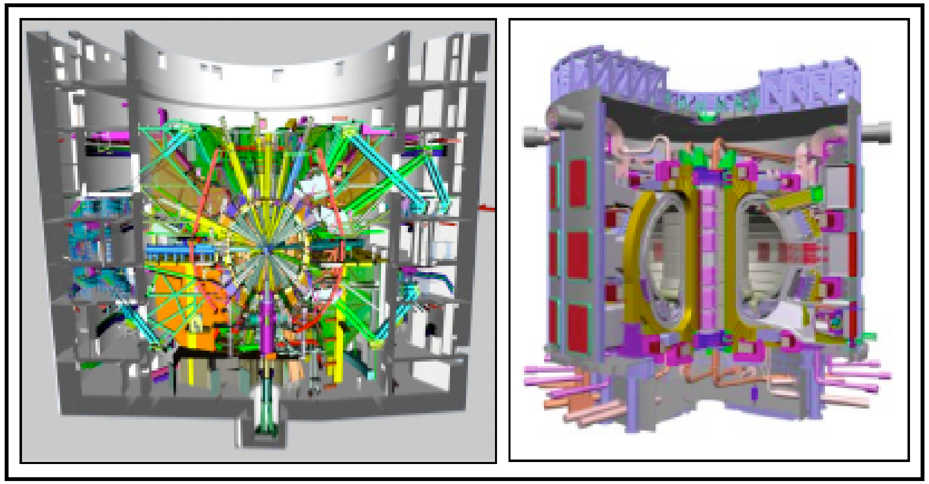
Context

-Astero and helioseismology spatial missions (**COROT**, MOST; **GOLF-NG (DYNAMICS)**, SOHO)

-Powerful ground-based instruments (ESPADONS, HARPS, VLT …)



-Physics instruments (LIL, LMJ, ITER)
→ Laboratory experiments relevant for astrophysical plasmas



-Numerical simulation of stellar (magneto-)hydrodynamics (ASH, ESTER)

→ **Dynamical vision of the Hertzsprung-Russel diagram in support of helio and asteroseismology**