The treatment of overshooting in stellar modelling: the need of a diffusive approach

Paolo Ventura

INAF - Osservatorio Astronomico di Roma - Italy

The natural way of dealing with convection would be to solve the Navier – Stokes equations..

$$\frac{\partial}{\partial t}\left(\rho u^i\right)+\nabla_k\left(\rho u^i u^k + g^{ik}P\right) = -\rho g^i + \nabla_k \sigma^{ik}(u)$$

Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla_k\left(\rho u^k\right) = 0$$

Energy conservation

$$\rho T\left(\frac{\partial S}{\partial t} + u^k \nabla_k S\right) = \rho\epsilon_n + \sigma^{ik}(u)\nabla_k u^i - \nabla_k F^k$$

Reynolds' approach: each variable is split into an average and a fluctuating part:

$$T = \overline{T} + \theta$$
$$\rho = \overline{\rho} + \rho'$$
$$v_z = \overline{v_z} + w$$

Multiplying the conservation equations for the variables involved, after averaging we get the equations for the second order momenta:

$$\frac{\partial}{\partial t}(\overline{w\theta}) + \frac{\partial}{\partial z}(\overline{w^2\theta} + \overline{P\theta}) = f(\overline{w^2}, \overline{\theta^2}, \overline{w\theta})$$

$$\frac{\partial}{\partial t}(\overline{\theta^2}) + \frac{\partial}{\partial z}(\overline{w\theta^2}) = g(\overline{w^2}, \overline{\theta^2}, \overline{w\theta})$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\overline{w^2}\right) + \frac{\partial}{\partial z}\left(\frac{1}{2}\overline{w^3} + \overline{Pw}\right) = h(\overline{w^2}, \overline{\theta^2}, \overline{w\theta}) - \frac{1}{3}\epsilon$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\overline{q^2}\right) + \frac{\partial}{\partial z}\left(\frac{1}{2}\overline{q^2 w} + \overline{Pw}\right) = g\alpha\overline{w\theta} - \epsilon$$

Convective flux

Temperature fluctuations

Kinetic energy (radial and total)

Dissipation

Non locality!

Convective zone: $\overline{w\theta}$ >0, $\nabla > \nabla_{ad}$

Overshooting region: Fc<0, $\nabla < \nabla_{ad}$

The extension of the overshooting region can be found from first principles by solving the above set of equations, finding the point where the kinetic energy becomes zero.

The solution of the equations is made difficult by the presence of the third order momenta: introducing further equation for these quantities would involve fourth order momenta, and so on..

The attempts made so far to deal with overshooting and, more generally, with convection, consist in solving the above equations by adopting some approximations.

$$\frac{\partial}{\partial z}(\overline{w^3}, \overline{wq^2}, \overline{\theta w^2}, \overline{w\theta^2}) = 0$$
$$\overline{\theta^2} = \epsilon = 0$$

→ $Fc = kL^2(\nabla - \nabla ad)^{3/2}$

MLT!

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\overline{q^2}\right) + \frac{\partial}{\partial z}\left(\frac{1}{2}\overline{q^2 w} + \cancel{\overline{Pw}}\right) = g\alpha\overline{w\theta} -$$

Roxburgh (1978)

Many authors investigated the effects of overshooting by adding some non-locality to one of the equations (e.f. Shaviv & Citre 1968; Bressan et al. 1981; Kuhfuss 1986; Gough 1976)

Xiong (1985)
Diffusive approximation for the third order momenta:

$$\overline{wq^2} = -\nu_t \frac{\partial}{\partial z}\overline{q^2}$$

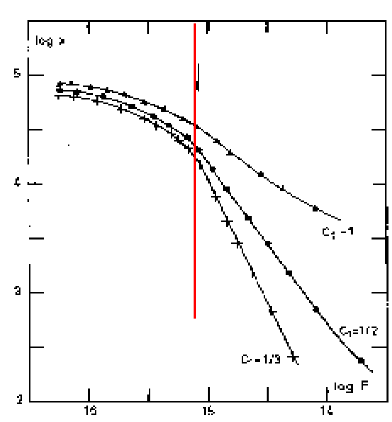
$$\overline{\theta^2 w} = -\nu_t \frac{\partial}{\partial z}\overline{\theta^2}$$

$$\overline{w^3} = -\nu_t \frac{\partial}{\partial z}\overline{w^2}$$

$$\overline{\theta w^2} = -\nu_t \frac{\partial}{\partial z}\overline{w\theta}$$

Parametrization of the scale!

$$\nu_t = C_1 w H_p$$



Convective velocity vs. Pressure (MS model)

Velocities decay exponentially out of the convective core; the scale length of decay depends on C_1!

Whatever approximation we use to solve the full set of equations, the results obtained in terms of the extension of the overshooting zone show the signature of the hypothesis made!!

Instantaneous overshooting

The extension L_{OV} of the extra-mixing region beyond the formal convective core is commonly parametrized by $L_{OV}=\alpha H_P$.

The whole convective zone (formal + overshoot) is assumed to be instantaneously homogenized by mixing, which is thus assumed to be much faster than any nuclear reaction.

CMDs provide an excellent tool to understand the extent of the extra-mixing region, at least for that concerning the convective core during the H-burning phase (Stothers 1991; Demarque et al. 1994; Maeder & Meynet 1987,89)

$L_{OV}=0.2H_P$ for $M>1.5M_{sun}$

$L_{OV}\rightarrow 0$ for $\rightarrow 1.1M_{sun}$

Diffusive extra-mixing

Conservation of species i

$$\frac{\partial \rho_i}{\partial t} + \nabla \rho_i u = \cancel{-\nabla J_i} + q_i$$

Molecular diffusion

Nuclear term

$$\rho_i = \overline{\rho_i} + \rho_i'$$

$$u = \overline{u} + u'$$

$$\Rightarrow \quad \frac{\partial \overline{\rho_i}}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2(\overline{\rho_i}\overline{u} + \overline{\rho_i' u'})] = q_i$$

Gradient approximation $\Rightarrow$

$$\overline{\rho_i' u'} = -D\frac{\partial \overline{\rho_i}}{\partial r}$$

$$\boxed{\frac{dX_i}{dt} = \frac{\overline{q_i}}{\overline{\rho}} + \frac{\partial}{\partial m}\left((4\pi r^2 \rho)^2 D \frac{dX_i}{dm}\right)}$$

The coefficient D would be known from first principles only by solving the NS equations; this renders mandatory to use a local approximation, based on dimensional arguments

$$D = 1/3\, v\, L$$

Convective velocity

Scale length

How do we model overshoot?

Based on numerical simulations (Freytag et al. 1996), and on theoretical results (e.g. Xiong 1985), we allow overshoot to vanish exponentially, by setting

$$D(r) = D_0 epx[-r/(f^* H_P)]$$

Physical meaning completely different from α !!

Positive and negative sides of the diffusive approach for overshooting

Good

- ✓ The efficiency of mixing vanishes far from the formal border
- ✓ Nuclear burning and mixing of chemicals are treated simultaneously, with no "a priori" assumptions

Bad

- ✓ A typical length scale must be provided
- ✓ The scale height of the exponential decay of velocities within the radiative regions is parametrized
- ✓ It is time consuming !!!

Stellar contexts where the diffusive approach is highly recommended

- ✓ Modelling super-rich lithium stars
- ✓ The core – He burning phase of intermediate mass stars, and the distribution of stars in the clump region of opne clusters
- ✓ The AGB evolution as a whole, and the formation of carbon stars

Lithium production in AGBs

Historically, one of the first mandatory use of the diffusive approach was to model the super-rich lithium stars, the presence of which was extensively confirmed by deep spectroscopic analysis of MCs luminous AGBs (Smith & Lambert 1989, 1990)

1) $^7Li+p \rightarrow 2\alpha$
2) $^7Be+e^- \rightarrow {}^7Li$
3) $^3He+\alpha \rightarrow {}^7Be$

Easily activated, even during the PMS phase

τ = 53 days

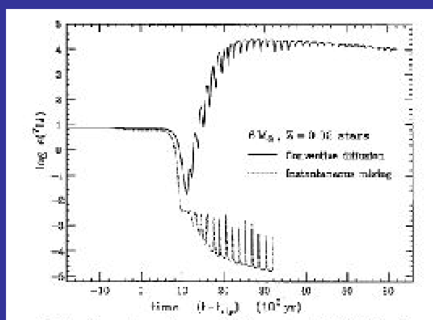
May be activated during the AGB phase when T exceeds 50×10^6 K

Lithium production is made possible during the AGB phase by the high temperatures achieved by the bottom of the convective envelope, which activate reaction (3). Otherwise, only proton fusion is possible, with no production of lithium

The mechanism for the production of lithium was correctly understood by Cameron & Fowler (1971). At the bottom of the envelope, the various time scales are related as follows:

$$\tau(^7\mathrm{Li}+p) << \tau_{mix} << \tau(^3\mathrm{He}+\alpha), \tau(\mathrm{Be\ decay})$$

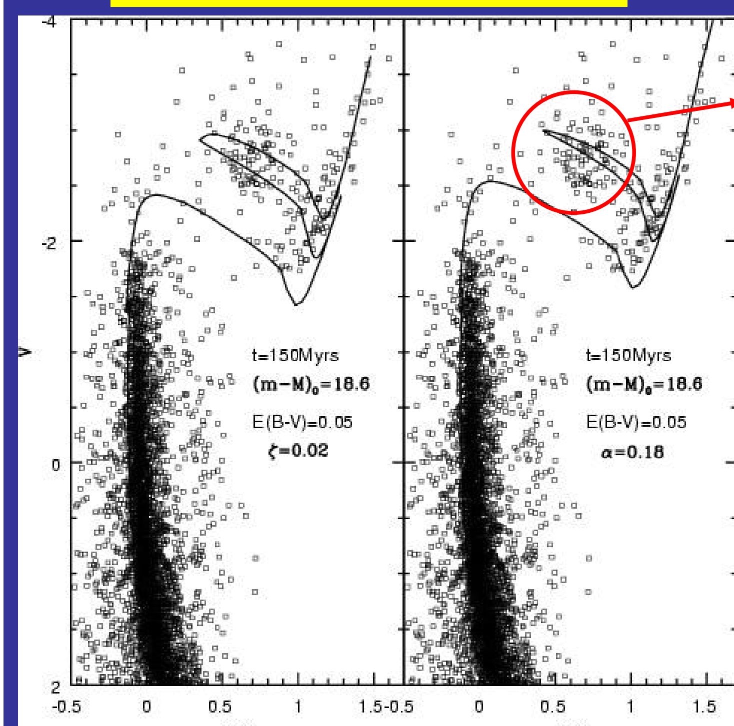
The Beryllium produced at the bottom of the envelope can be partially transported outwards before decaying into lithium; this latter is produced in outer (and thus cooler) layers, and can therefore survive to proton fusion



LogN(Li) vs. time (Sackmann & Boothroyd 1979)

No way of getting lithium production with the instantaneous mixing scheme, because some nuclear processes are faster than convection!!

CMD of NGC 1866



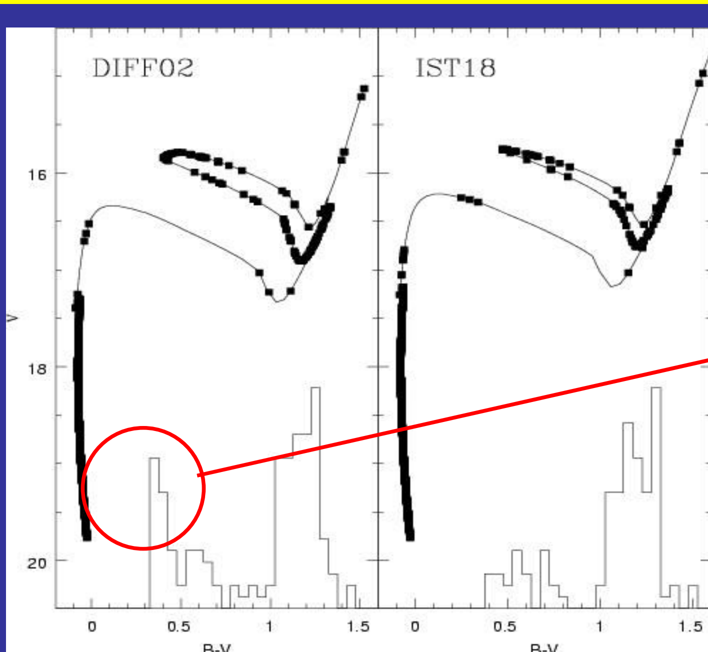
Testa et al. (1999)

The Clump of NGC 1866 shows a considerable component of stars burning Helium in the blue, with B/R=1

Fitting the luminosity of the clump and TO requires some overshooting from the core during MS phase

The mass currently in the core He-burning phase is 4-4.5 M_{sun}

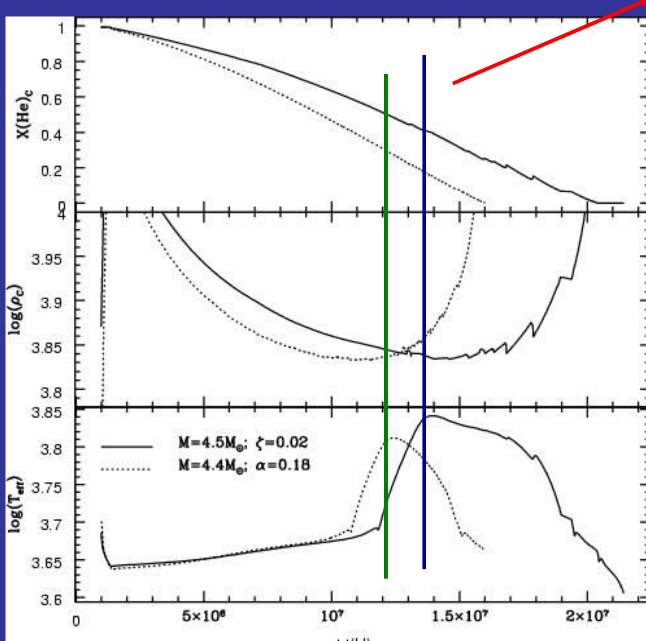
Models with either instantaneous or diffusive overshoot may well mimick the CMD but… what about the color distribution of clump stars?



When overshoot is modeled as a diffusive process, the simulations provide a fraction of blue clump stars which is consistent with the observational evidence

Ventura & Castellani (2005)

Core He-burning phase: diffusive vs. instantaneous approach

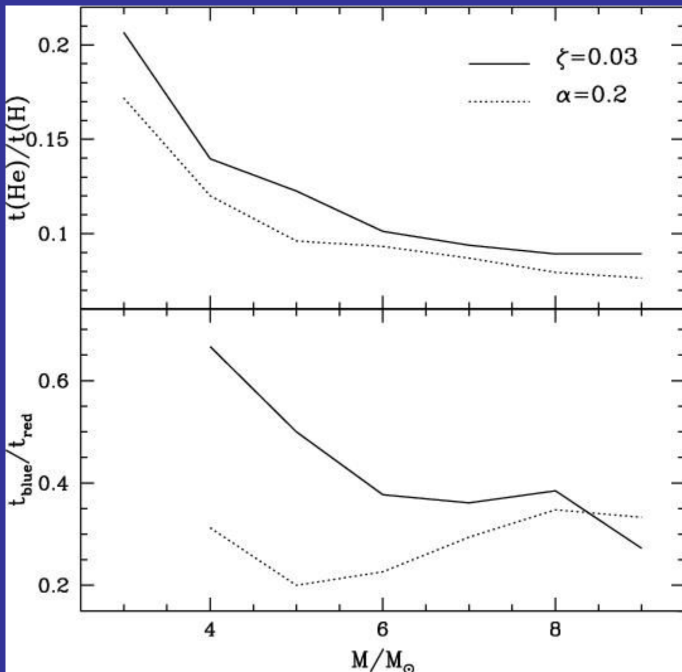


In the diffusive case helium burning is slower, due to the different scheme adopted

By the time that the bluest point is reached, the central helium abundance of the instantaneous model dropped to Y=0.2, so that the core contracts, and the star expands.

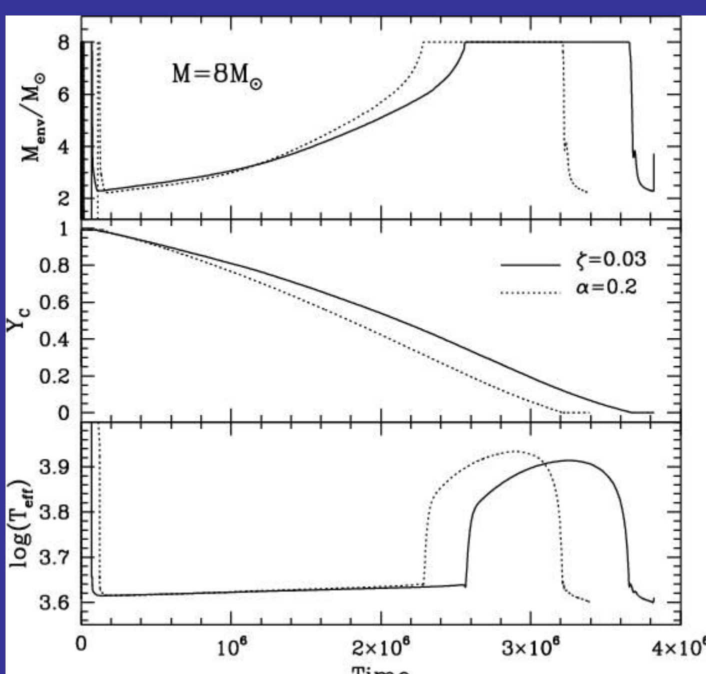
In the diffusive case helium burning is slower, thus the duration of the blue phase is longer

What happens for the other masses ?



The difference in the B/R ratio obtained with the two schemes for overshooting tens to vanish for larger masses. At M=8Msun, the relative duration of the blue phase is the same for the two models

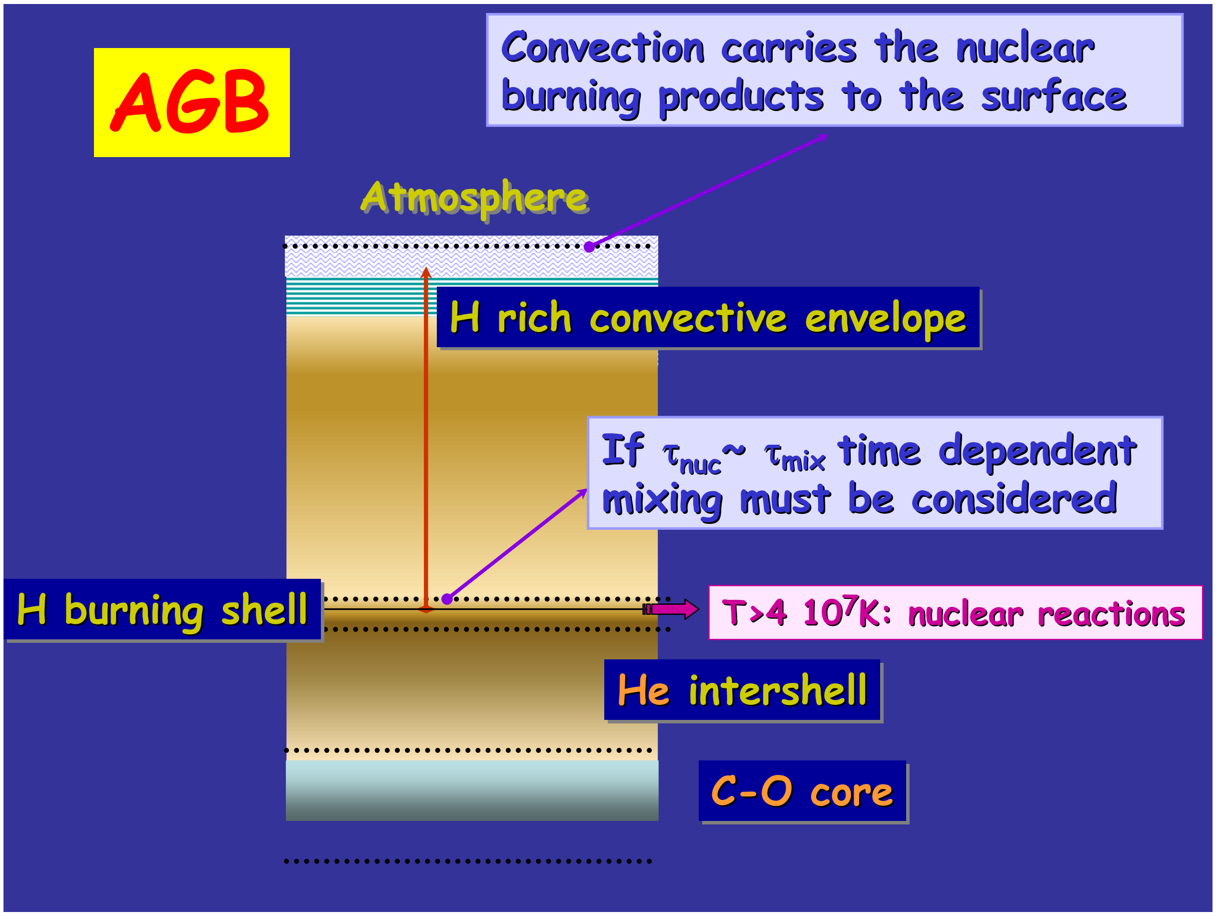
Ventura, Castellani & Straka (2005)



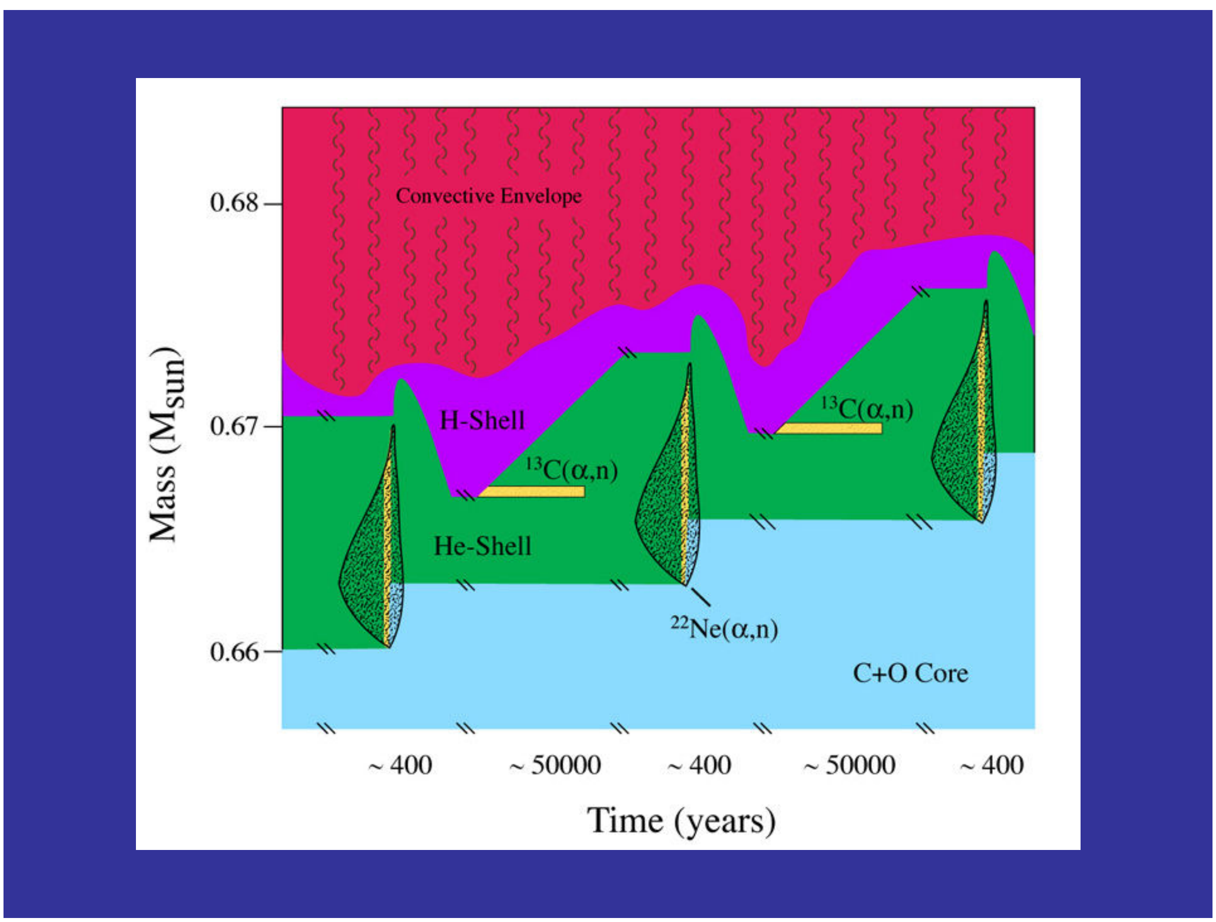
For larger masses, the differences tend to dimish because even the diffusive models reach the bluest point of the track with a low central Y

Adopting a diffusive approach is thus mandatory to investigate the core He-burning phase of models of intermediate masses, expecially for 3.5 <M/Msun < 6

Ventura, Castellani & Straka (2005)



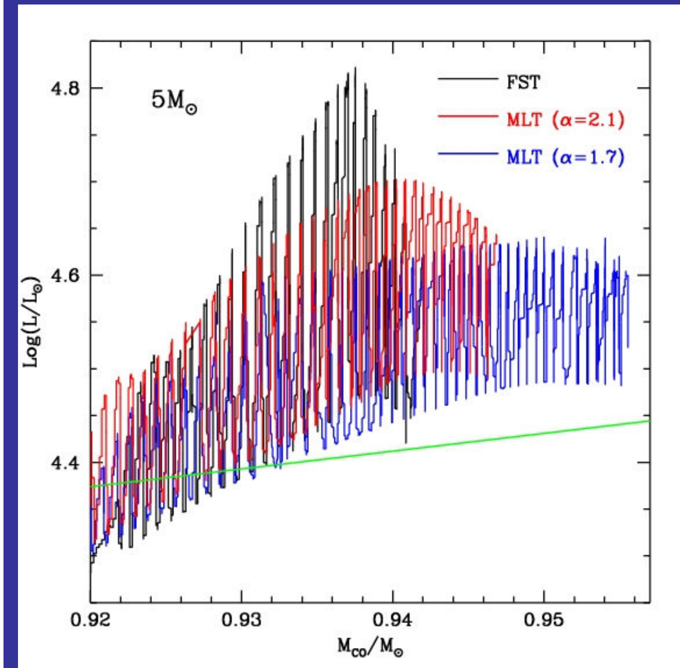
AGB
Convection carries the nuclear burning products to the surface
Atmosphere
H rich convective envelope
If $\tau_{nuc} \sim \tau_{mix}$ time dependent mixing must be considered
H burning shell
$T > 4\ 10^7 K$: nuclear reactions
He intershell
C-O core



Convective Envelope
H-Shell
$^{13}C(\alpha,n)$
$^{13}C(\alpha,n)$
He-Shell
$^{22}Ne(\alpha,n)$
C+O Core
0.68
0.67
0.66
Mass (M_{sun})
~ 400
~ 50000
~ 400
~ 50000
~ 400
Time (years)

AGB models are expected to follow the Paczynski (1970) relation linking luminosity to the core mass

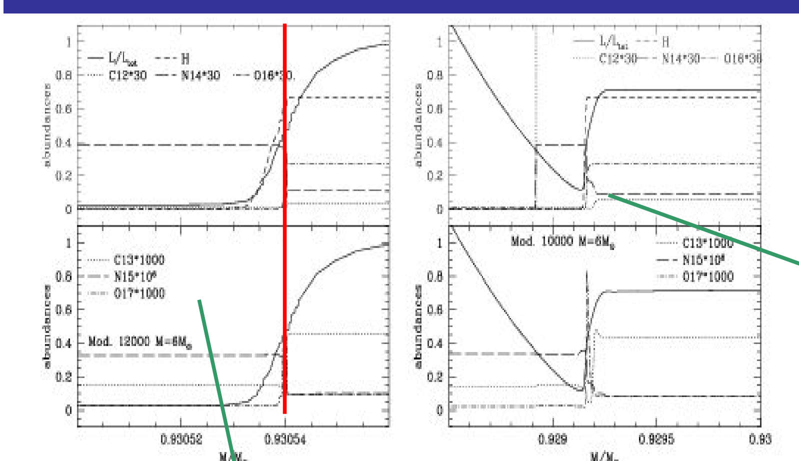
$$L=59250*(M_c/M_{sun}-0.522)$$



Bloecker & Schonberner (1991)
"The relation does not hold above a certain luminosity provided convection is efficient (L/Hp=2 in our case)"

The Paczynski relation demands a radiative buffer separating the CNO shell from the envelope!

AGB evolution of a popI 6Msun model



The physics of the post-TP phase is highly uncertain, particularly for that concerning the possible extension of the 3rd dredge-up. No way of using any instantaneous scheme here..

During the interpulse phase in the most massive models almost 30% of the luminosity is generated directly within the envelope: the luminosity depends on the details of the CNO abundances, so the diffusive treatment is recommended!