# ADIPACK

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## The Aarhus Stellar Pulsation Code

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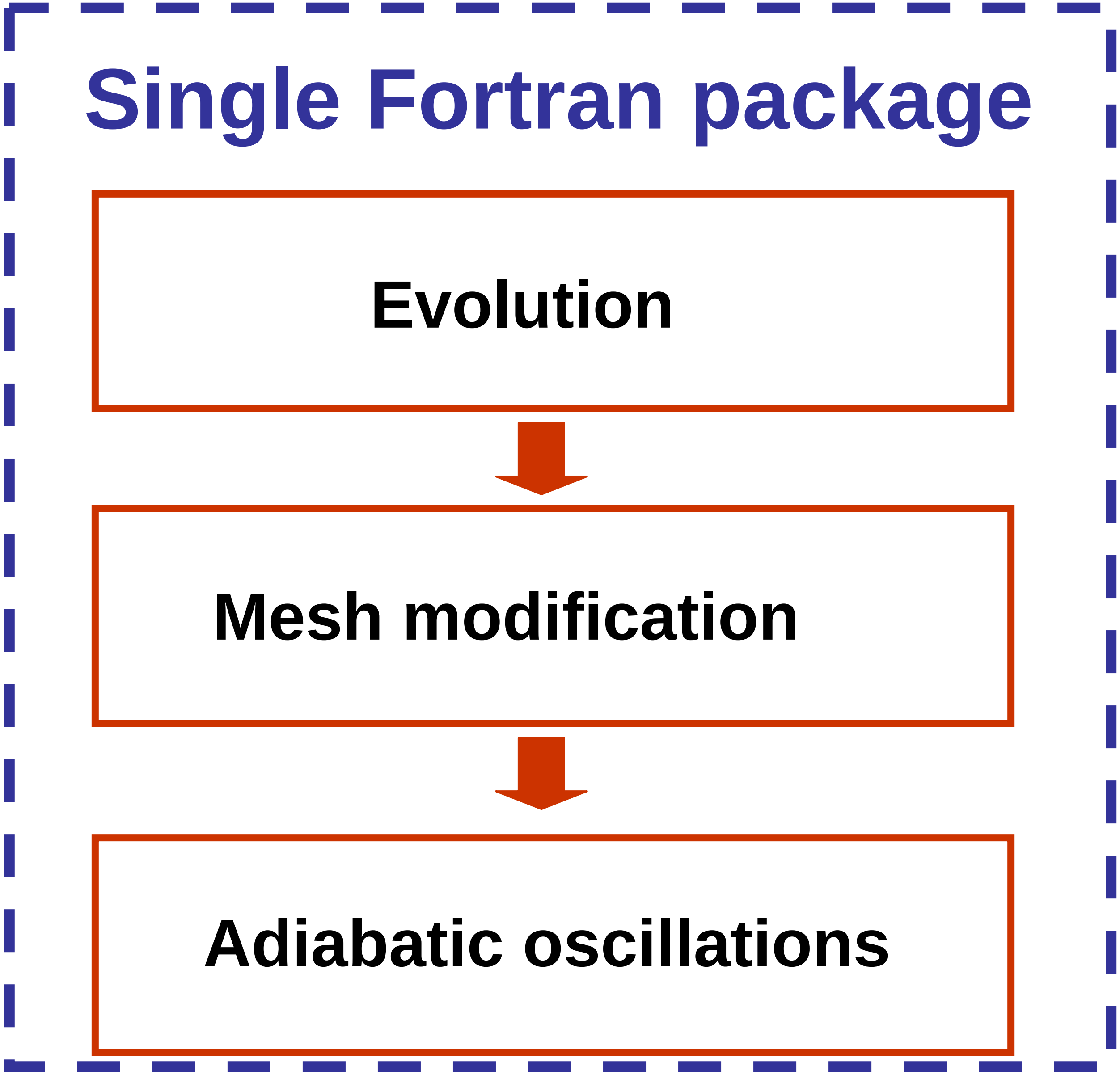
# Overall structure



# Overview

- Adiabatic equations, full case and Cowling approximation
- Solution with second-order difference scheme
- Improved frequencies using Richardson extrapolation or variational principle
- Fourth-order difference scheme implemented but not carefully tested.

# Equations

$$x\frac{\mathrm{d}y_1}{\mathrm{d}x} = (V_g - 2)y_1 + \left(1 - \frac{V_g}{\eta}\right) y_2 - V_g y_3 \ ,$$

$$x\frac{\mathrm{d}y_2}{\mathrm{d}x} = [l(l+1) - \eta A]y_1 + (A-1)y_2 + \eta A y_3 \ ,$$

$$x\frac{\mathrm{d}y_3}{\mathrm{d}x} = y_3 + y_4 \ ,$$

$$x\frac{\mathrm{d}y_4}{\mathrm{d}x} = -AUy_1 - U\frac{V_g}{\eta}y_2$$

$$+[l(l+1) + U(A-2) + UV_g]y_3 + 2(1-U)y_4 \ .$$

$$y_1 = \frac{\xi_r}{R} \ , \quad y_2 = x\left(\frac{p'}{\rho} - \Phi'\right)\frac{l(l+1)}{\omega^2 r^2} = \frac{l(l+1)}{R}\xi_h \ ,$$

$$y_3 = x\frac{\Phi'}{gr} \ , \quad y_4 = x^2 \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y_3}{x}\right)$$

# Boundary conditions

- Regularity conditions at centre
- Match to solution for isothermal atmosphere at surface; or
- Vanishing Lagrangian pressure perturbation
- In full polytropic models regularity condition at surface

# Finding the eigenfrequencies

- Integrate two solutions from centre or surface, find zero of matching determinant; or
- Impose boundary conditions; require continuity of solution at matching point.
- Trial frequencies from previous solution; or
- Scan in frequency to locate eigenfrequencies

# Improving the frequencies

Richardson extrapolation:

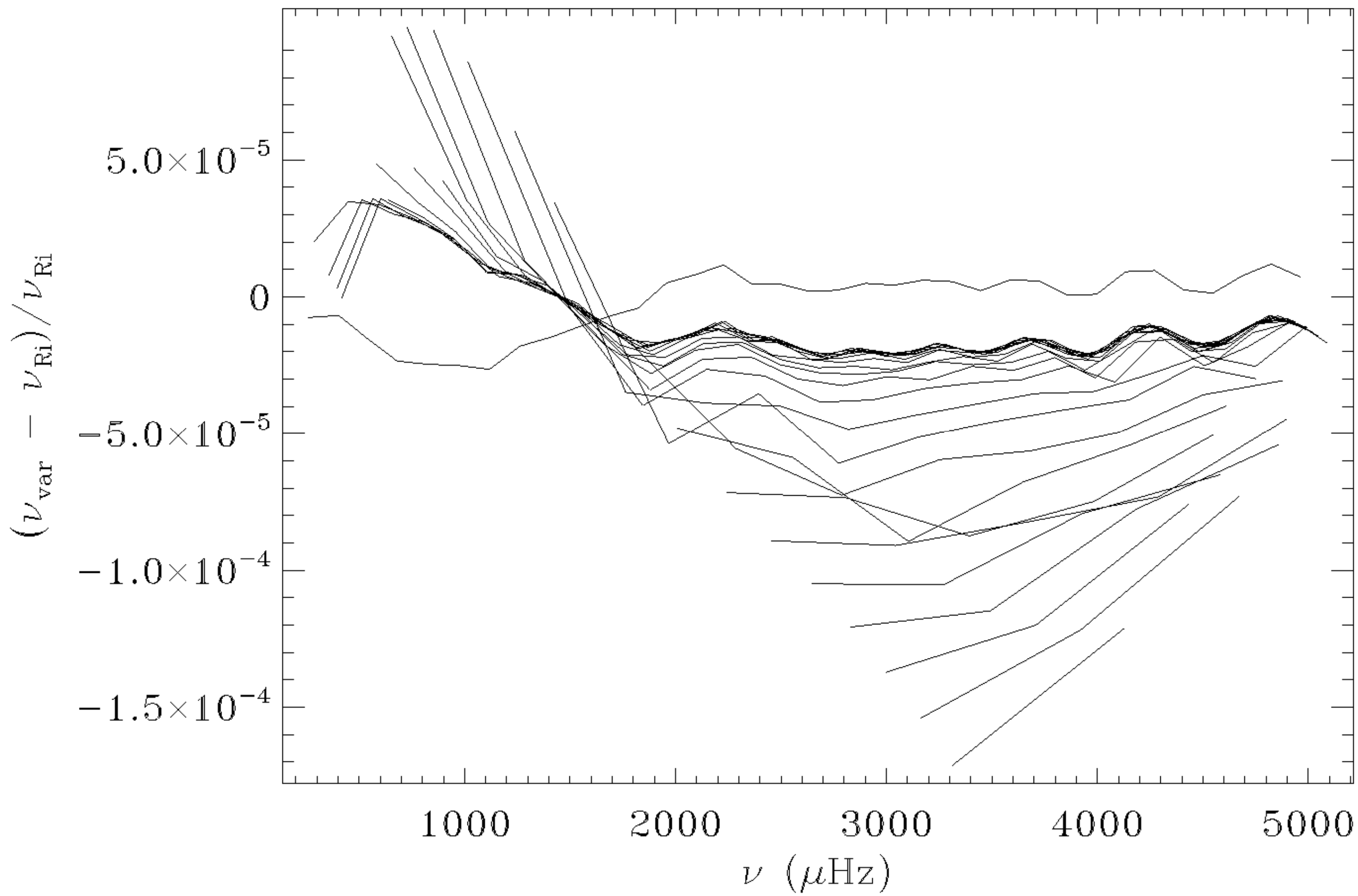
$$\omega_{\mathrm{Ri}} = \frac{1}{3}[4\omega(N) - \omega(N/2)]$$

Variational frequency:

$$\omega^2 = \frac{\int_V \xi \mathcal{L} \xi \rho \mathrm{d}V}{\int_V |\xi|^2 \rho \mathrm{d}V}$$

**Note: difference between the two values tests (in)consistency of model**

# Example: Model S



# Implementation details: Pulsation mesh

Mesh reset for pulsation calculation, depending on desired modes (distribution largely determined by asymptotic behaviour of eigenfunctions).

Let $x = r/R$, $\hat{c}^2 = (R/GM)c^2$, $\hat{N}^2 = (R^3/GM)N^2$.

$$g(x) = \left[1 + w_1^2 \hat{c}^{-2} + w_2^2 \frac{|\hat{N}|^2}{x^2} + w_3^2 \left(\frac{\mathrm{d}\ln p}{\mathrm{d}x}\right)^2\right]^{1/2}$$

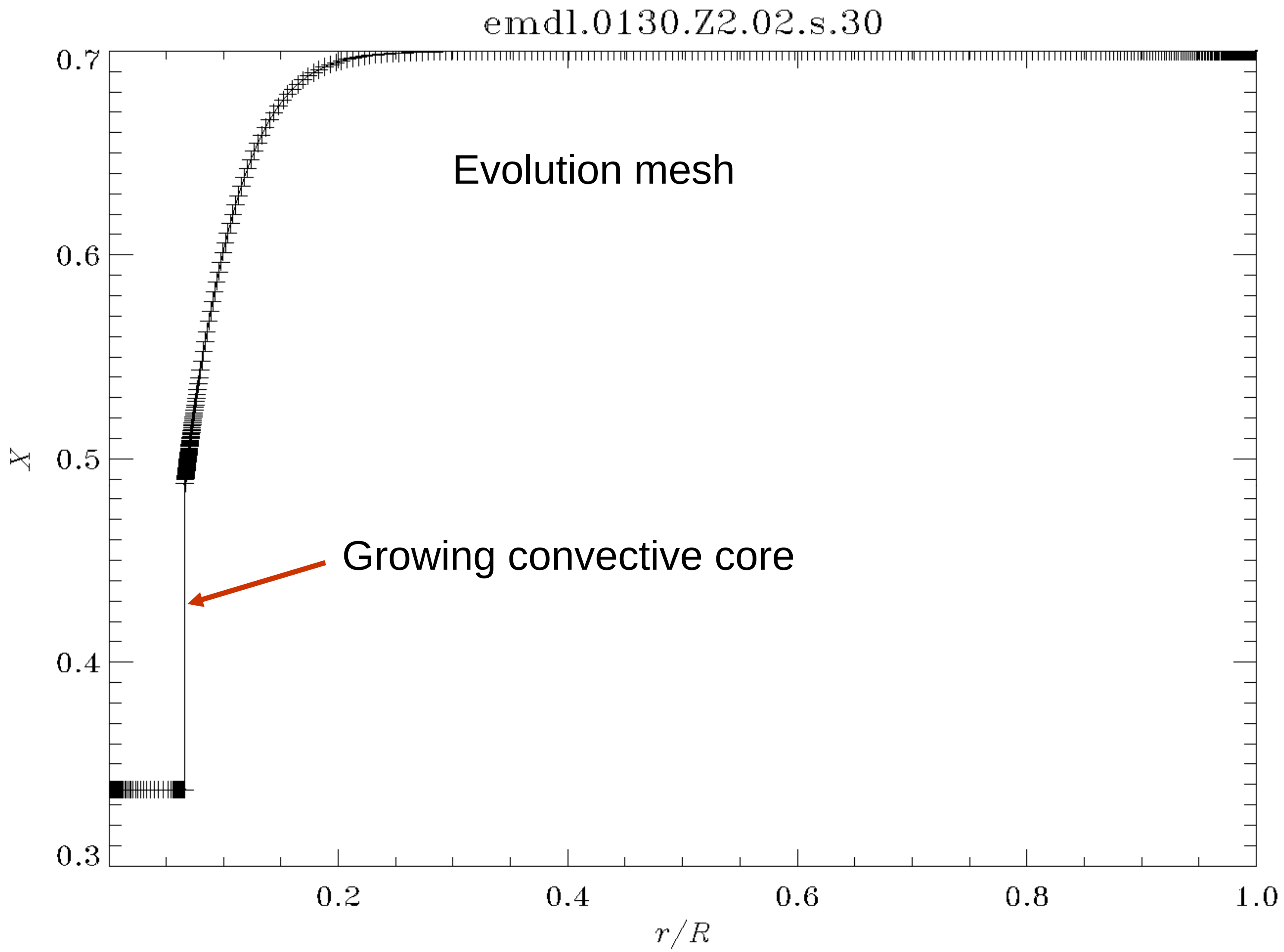
Note that if term in $w_1$ dominates, mesh is uniform in

$$\int_0^x \frac{\mathrm{d}x}{\hat{c}} ,$$

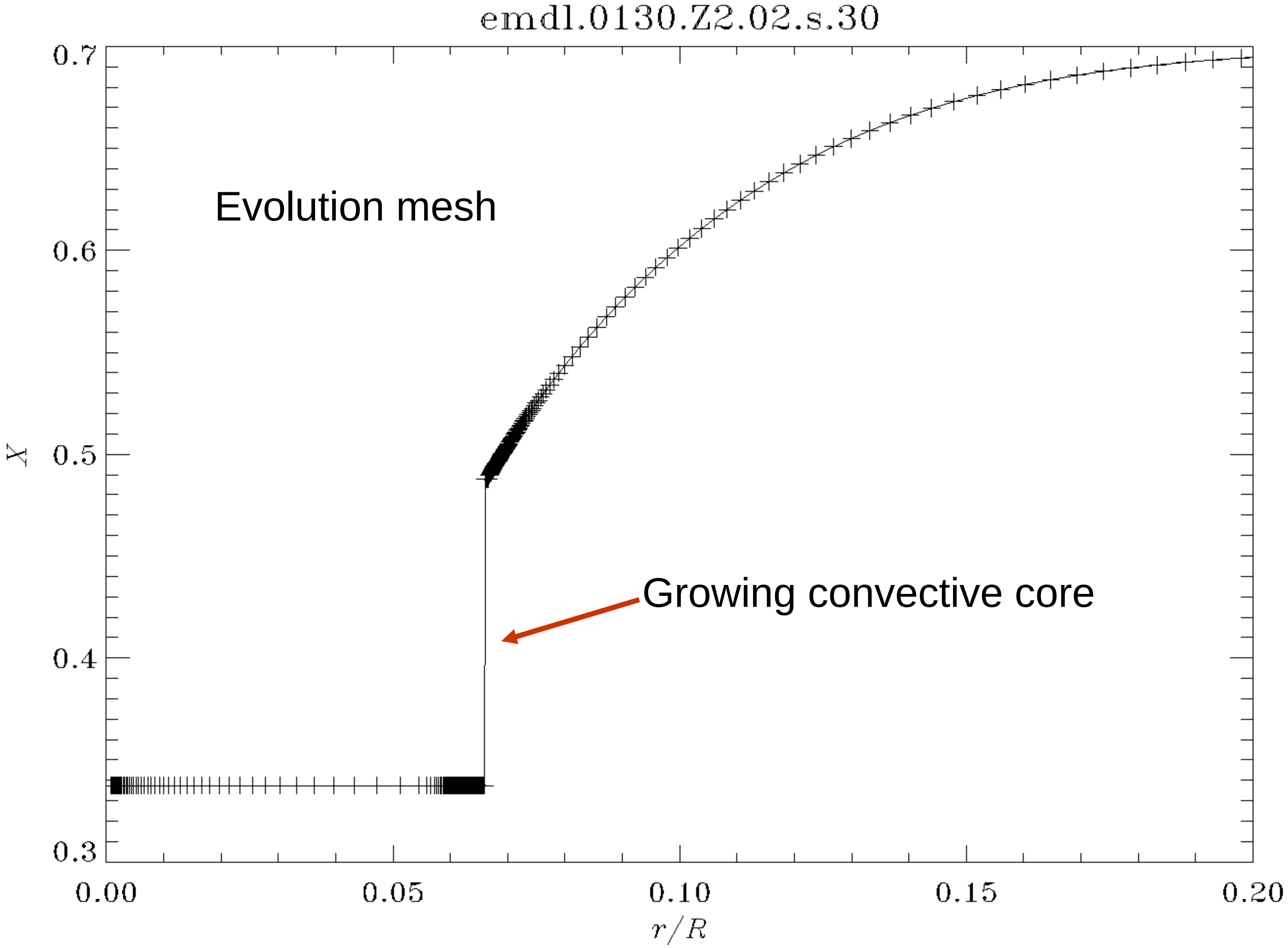
i.e., acoustical radius, as is appropriate for acoustic modes.

Additional points are added near discontinuities in density.

# 1.3 $M_\odot$

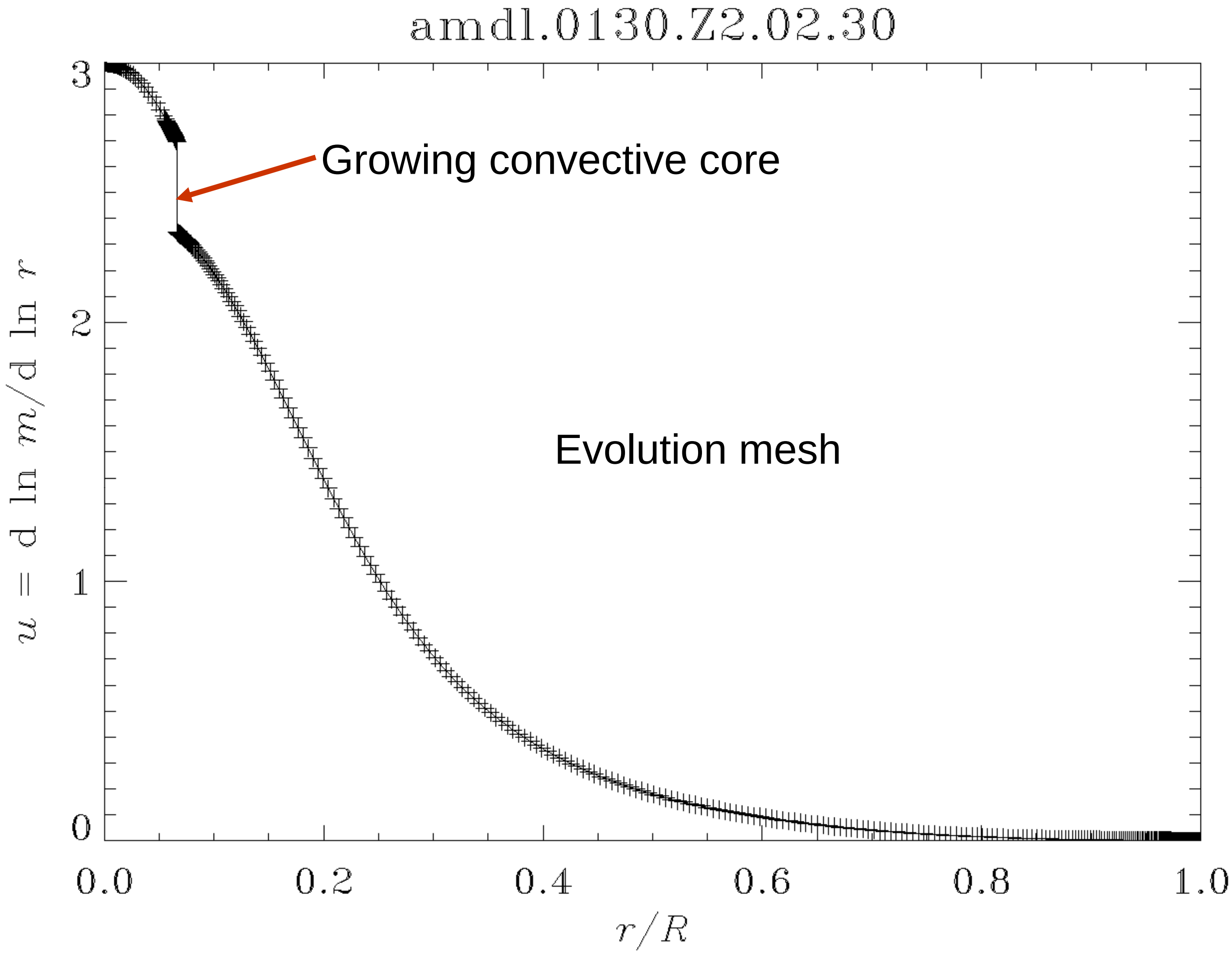

emdl.0130.Z2.02.s.30
0.7
0.6
0.5
0.4
0.3
X
0.2
0.4
0.6
0.8
1.0
r/R
Evolution mesh
Growing convective core

# 1.3 $M_\odot$

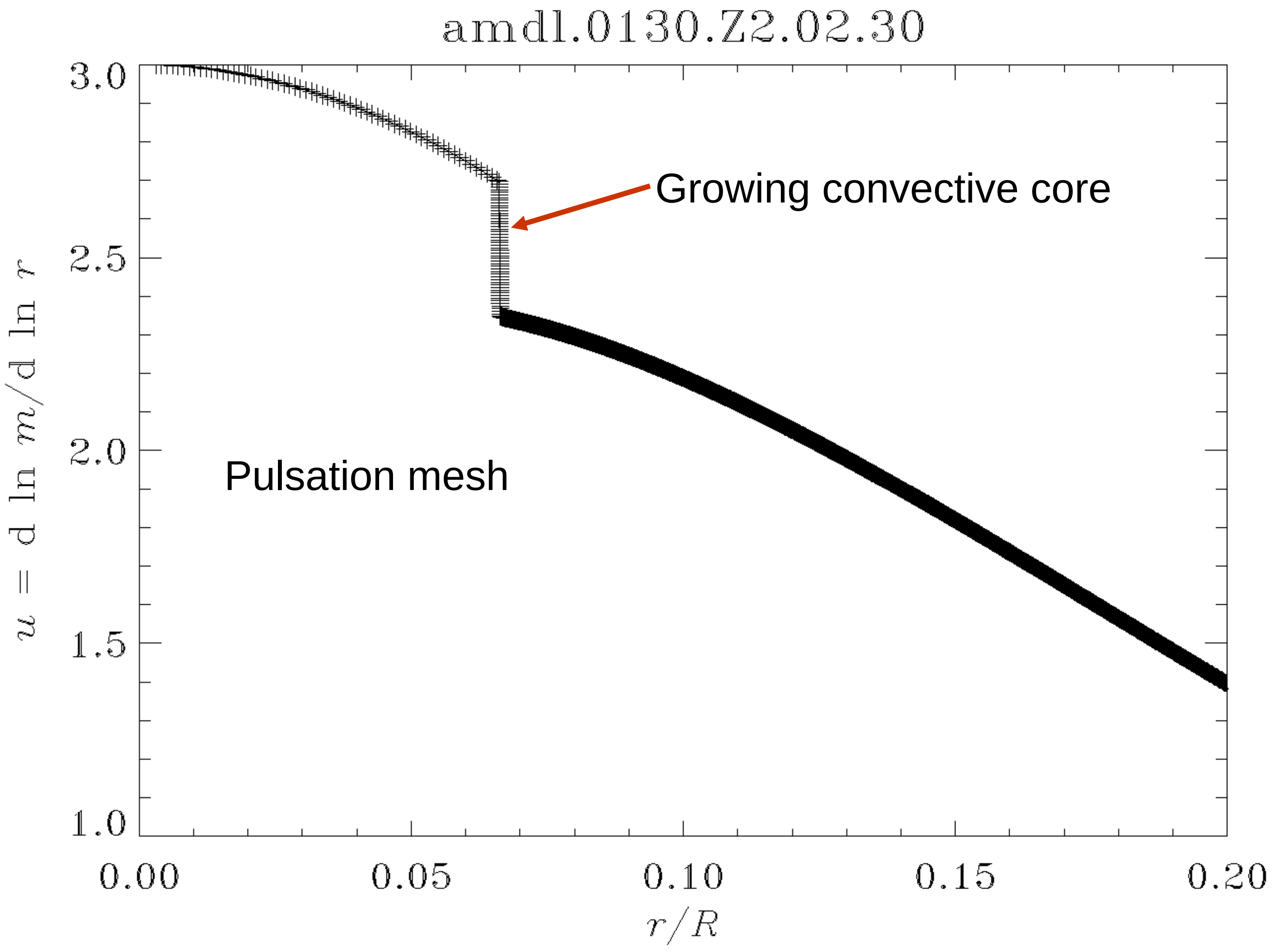


emdl.0130.Z2.02.s.30
Evolution mesh
Growing convective core
0.7
0.6
0.5
0.4
0.3
X
0.00
0.05
0.10
0.15
0.20
r/R

# 1.3 $M_\odot$

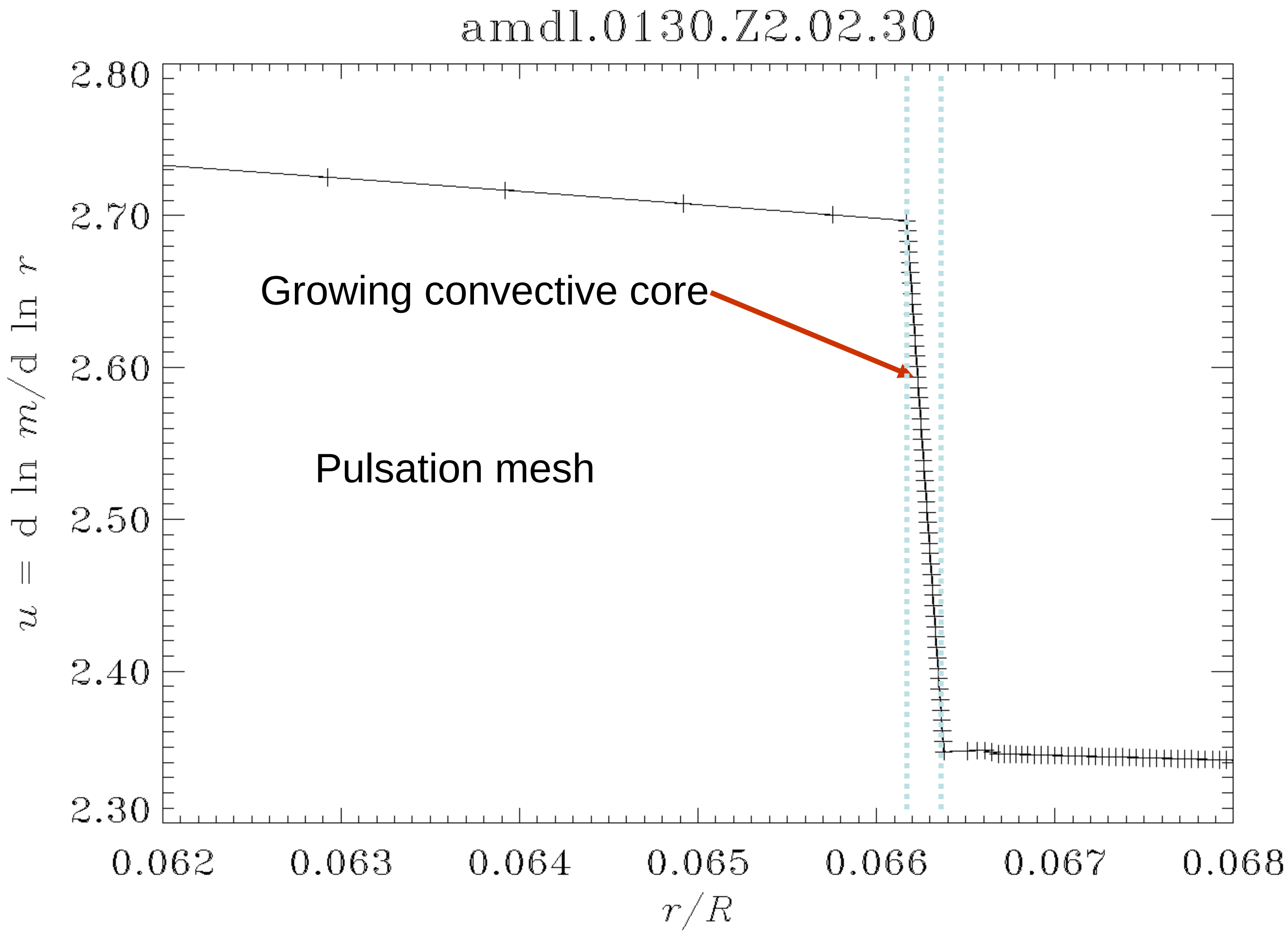

amdl.0130.Z2.02.30
Growing convective core
Evolution mesh
$u$ = d ln $m$/d ln $r$
$r/R$
0
1
2
3
0.0
0.2
0.4
0.6
0.8
1.0

# 1.3 $M_\odot$



amdl.0130.Z2.02.30
Growing convective core
Pulsation mesh
$u = \mathrm{d}\ln m / \mathrm{d}\ln r$
$r/R$
3.0
2.5
2.0
1.5
1.0
0.00
0.05
0.10
0.15
0.20

# 1.3 $M_\odot$


amdl.0130.Z2.02.30
2.80
2.70
2.60
2.50
2.40
2.30
$u$ = d ln $m$/d ln $r$
0.062
0.063
0.064
0.065
0.066
0.067
0.068
$r/R$
Growing convective core
Pulsation mesh