The Liège Oscillation Code

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The Liège Oscillation Code

1. Inputs

Inputs

- global M and R
- radius: r
- mass: m(r)
- density: ρ (r)
- pressure: P(r)
- adiabatic: $\Gamma_1(r)$

dimensionless variables:

- $x = r/R$,
- $q/x3$ (with $q=m/M$)
- $RP/GM\rho$,
- $4\pi R^3 \rho/M$
- Γ_1

2. Non-radial adiabatic oscillations

The perturbation is described by 4 functions $Y(x)\ Z(x)\ U(x)\ V(x)$

$$Y(x) = x^{1-\ell}\,\frac{\delta r}{R}$$

$$Z(x) = x^{-\ell}\,\frac{\delta P}{P}$$

$$U(x) = x^{-\ell}\,\frac{R\,\Phi'}{G\,M}$$

$$V(x) = x^{1-\ell}\left(\frac{R^2}{G\,M}\,\frac{d\,\Phi'}{dr} + \frac{4\,\pi\,\rho\,R^3}{M}\,\frac{\delta\,r}{R}\right).$$

2. Non-radial adiabatic oscillations

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$$Y(x) = x^{1-\ell}\,\frac{\delta r}{R}$$

Lagrangian perturbation of the pressure

$$Z(x) = x^{-\ell}\,\frac{\delta P}{P}$$

Eulerian perturbation of the Gravitational potential

$$U(x) = x^{-\ell}\,\frac{R\,\Phi'}{G\,M}$$

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2. Non-radial adiabatic oscillations

A nonradial mode :

- $$\vec{\delta r} = \sqrt{4\pi}\Re\left\{\left[a(r)Y_{\ell m}(\theta,\phi)\vec{e}_r + b(r)\left(\frac{\partial Y_{\ell m}}{\partial\theta}\vec{e}_\theta + \frac{1}{\sin\theta}\frac{\partial Y_{\ell m}}{\partial\phi}\vec{e}_\phi\right)\right]e^{-i\sigma t}\right\}$$

$$a(r)/R = x^{\ell-1}Y(x) \qquad b(r)/R = \frac{x^{\ell-1}}{\omega^2}\left[U(x) + \frac{RP}{GM\rho}Z(x) + \frac{q}{x^3}Y(x)\right]$$

- $$\frac{\delta P}{P} = \sqrt{4\pi}\Re\{x^\ell Z(x)Y_{\ell m}(\theta,\phi)e^{-i\sigma t}\}$$
- $$\frac{R\Phi'}{GM} = \sqrt{4\pi}\Re\{x^\ell U(x)Y_{\ell m}(\theta,\phi)e^{-i\sigma t}\}$$
- $$\frac{R^2}{GM}\frac{\partial\Phi'}{\partial r} = \sqrt{4\pi}\Re\left\{x^{\ell-1}\left[V(x) - \frac{4\pi R^3\rho}{M}Y(x)\right]Y_{\ell m}(\theta,\phi)e^{-i\sigma t}\right\}$$

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2. Non-radial adiabatic oscillations

Differential equations

- $$\frac{dY}{dx} = \frac{\ell+1}{x}\left\{-Y + \frac{\ell}{\omega^2}\left(\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right)\right\} - \frac{x}{\Gamma_1}Z$$
- $$\frac{dZ}{dx} = \frac{GM\rho}{RP}\left\{\left(\omega^2 + 4\frac{q}{x^3}\right)\frac{Y}{x} + x\frac{q}{x^3}Z - \frac{V}{x} - \frac{\ell(\ell+1)}{x\omega^2}\frac{q}{x^3}\left(\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right)\right\} - \frac{\ell}{x}Z$$
- $$\frac{dU}{dx} = \frac{1}{x}\left(V - \frac{4\pi R^3\rho}{M}Y - \ell U\right)$$
- $$\frac{dV}{dx} = \frac{\ell+1}{x}(\ell U - V) + \frac{\ell(\ell+1)}{x\omega^2}\frac{4\pi R^3\rho}{M}\left(\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right)$$

2. Non-radial adiabatic oscillations

- Boundary conditons at the centre: regularity

$$Y = \frac{\ell}{\omega^2}\left[\frac{q}{x^3}Y + \frac{RP}{GM\rho}Z + U\right]$$

$$V = \frac{4\pi R^3\rho}{M}Y + \ell U$$

- Boundary conditions at the surface
  - Gravitation potential $V + (\ell+1)U = 0$
  - Pressure $\frac{\delta P}{P} + (4+\omega^2)\frac{dr}{r} = 0$: Standard option in LOSC

    $\delta P = 0$

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3. Numerical thecniques

3.1 Determination of mesh:

Option: Grid as based on eigenfunction asymptotic behaviour

3.2 Difference equations: 4th order scheme

We do not improve the solution with the Richardson extrapolation method

3.3 Inverse Iteration method (Keeley 1977, ApJ211,926)

To solve the eigenvalue problem $(A - \lambda B)\vec{y} = 0$ with $\lambda = \omega^2$

3.2 Difference equations

4th order scheme based on the identity:

$$\vec{y}_i + \frac{h}{2}\vec{y}_i' + \frac{h^2}{12}\vec{y}_i'' = \vec{y}_{i+1} - \frac{h}{2}\vec{y}_{i+1}' + \frac{h^2}{12}\vec{y}_{i+1}'' + O(h^5)$$

with $h = x_{i+1} - x_i$

If $\vec{y}$ satisfies $\dfrac{d\vec{y}}{dx} = A(x)\vec{y}$

$$\left\{1 + \frac{h}{2}\alpha_i + \frac{h^2}{12}\beta_i\right\}\vec{y}_i = \left\{1 - \frac{h}{2}\alpha_{i+1} + \frac{h^2}{12}\beta_{i+1}\right\}\vec{y}_{i+1}$$

$$\alpha = A$$

$$\beta = A^2 + \frac{dA}{dx}$$

4. Outputs

- angular (σ) and dimensionless angular (ω) frequency
- mode order (k):
    - Phase diagramme : Scuflaire (1974, A&A 36, 107)
    - Lee (1985, PASJ 37,279) for more condensed stars

    k= 0,1,2,… for p-modes with l>1

    k= 1,2,3… for p-modes with l=0,1

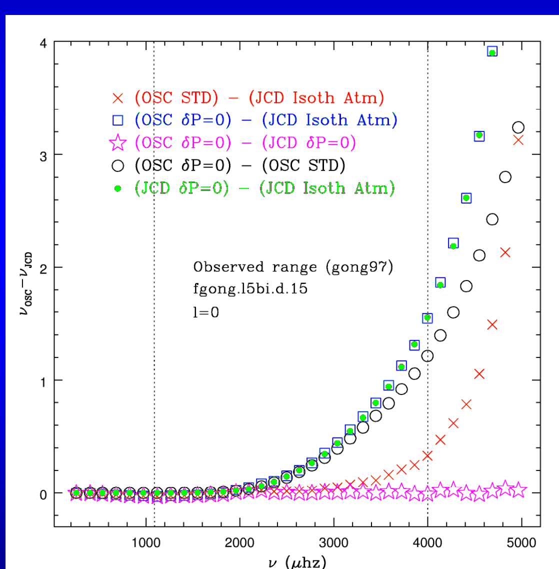
    k= -1,-2,-3… for g-modes
- mode parity

4. Outputs

- $f_T = E_{kin,V} / E_{kin}$
- $\langle x \rangle$
- $\Delta = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$
- rotational splitting coeffcient $\sigma_{k\ell m} = \sigma^0_{k\ell} + m\beta_{k\ell}\Omega$
- kernels $\sigma_{k\ell m} = \sigma^0_{k\ell} + m \int K_{k\ell}(x)\Omega(x)dx$
- Eigenfunctions: $Y(x)\ Z(x)\ U(x)\ V(x)$

  and $Y'(x)\ Z'(x)\ U'(x)\ V'(x)$

5. Comparison with ADIPLS



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