Modeling stellar interiors with rotational mixing

What has been achieved ?

What needs to be done ?

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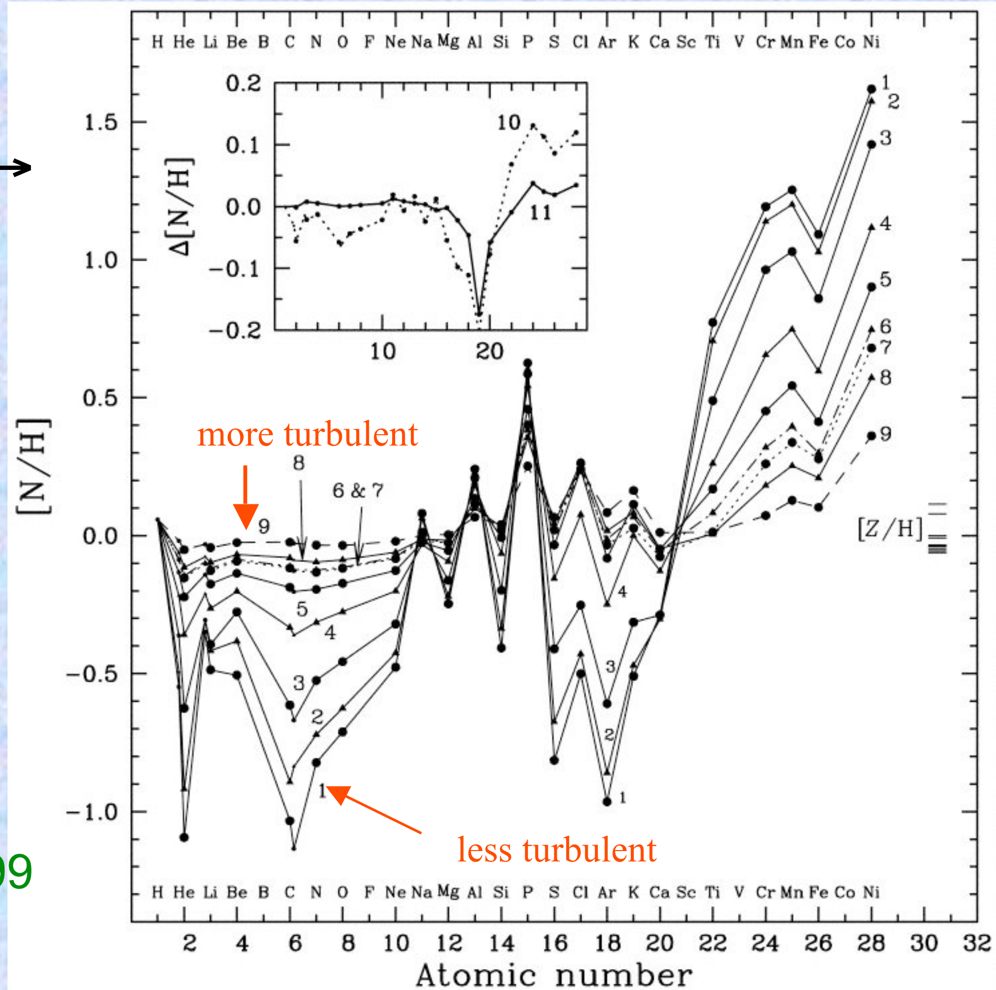
Modeling stellar interiors: a brief history

- 50's: first realistic models Schwarzschild criterion, MLT
  [Boehm-Vitense, Schwarzschild & coll.]
- 60's: evolutionary sequences
  [Cameron & Ezer, Hofmeister, Kippenhahn & Weigert, Iben]
- 70's: introduction of overshoot various parametrizations
  [Maeder, Roxburgh, etc.]
- 80's: attempts of introducing turbulent mixing crude parametrizations
  [Maeder, Schatzman, Sofia & coll., JPZ, etc.]
- 90's: implementation of microscopic diffusion
  + gravitational settling + radiative acceleration
  [Michaud, Vauclair, & coll.]

→ **PRESENT STANDARD MODEL**

PRESENT STANDARD MODEL

surface abundances in A type star →



Without mixing, which partly destroys
the effect of radiative acceleration
and gravitational settling,
tepid stars would exhibit
strong differences
in their surface composition

Schatzman 1969, Michaud et al 1999

→ evidence of extra mixing

How to treat this extra mixing

Minimalist approach

Introduce a parametrized turbulent diffusivity
Adjust parameters to fit observations

Parametrized turbulent diffusion

Fe and Li in NGC 6397

best fit with turbulent diffusion coefficient

$D_t = 400\ D(\text{mol, He, T}=10^6\text{K})\ (\rho_0/\rho)^3$

Korn et al. 2006

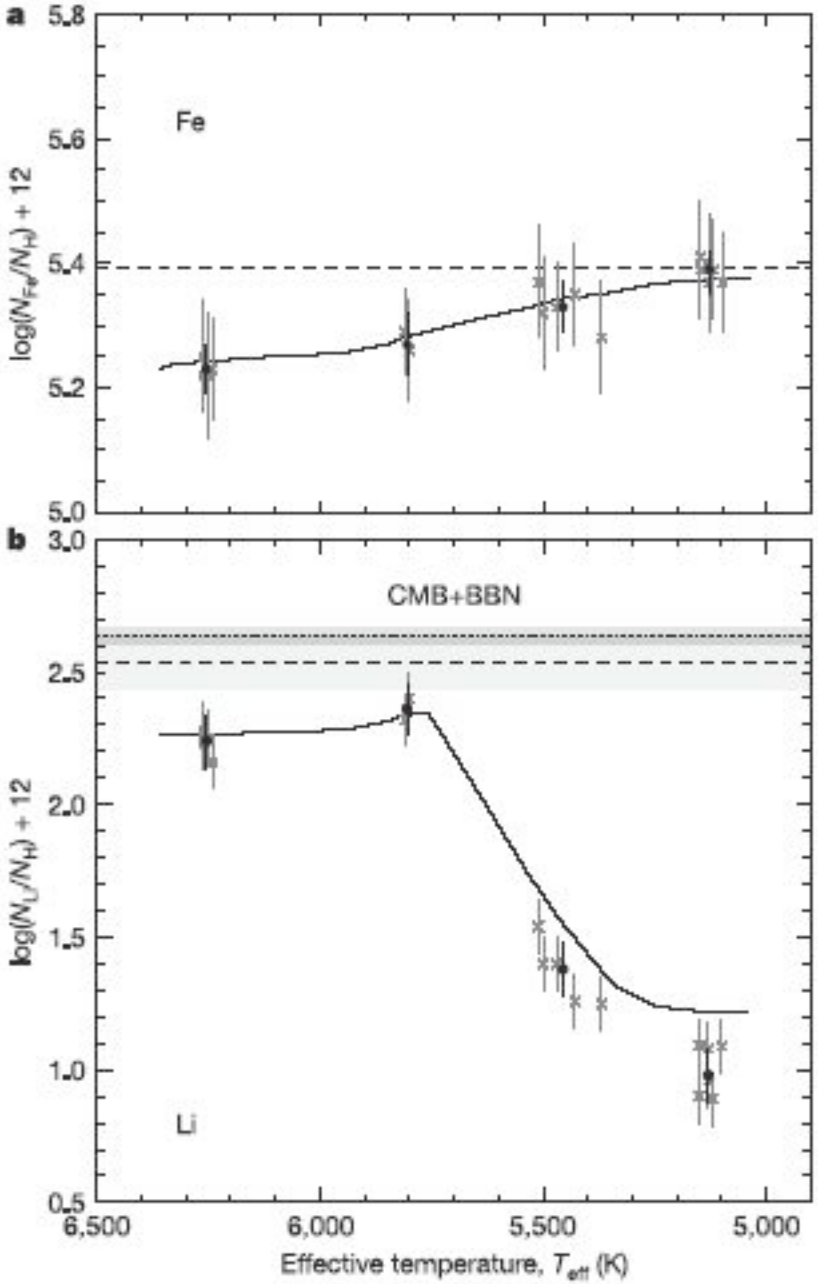


Figure 1 | Trends of iron and lithium as a function of the effective temperatures of the observed stars compared to the model predictions.

How to treat this extra mixing

Minimalist approach

Introduce a parametrized turbulent diffusivity
Adjust parameters to fit observations

Physical approach

Strive to implement the physical processes
that are likely to cause mixing

- turbulence produced by instabilities (shear, etc.)
- meridional circulation

Mixing processes in radiation zones

Meridional circulation

Classical picture: circulation is due to thermal imbalance
caused by perturbing force (centrifugal, magn. field, etc.)
Eddington (1925), Vogt (1925), Sweet (1950), etc

Eddington-Sweet time $$t_{\rm ES} = t_{\rm KH}\frac{GM}{\Omega^2 R^3}, \quad \text{with } t_{\rm KH} = \frac{GM^2}{RL}$$

Revised picture: after a transient phase of about t_{ES},
circulation is driven by the loss (or gain) of angular momentum
and structural changes due to evolution
Busse (1981), JPZ (1992), Maeder & JPZ (1998)

- no AM loss: no need to transport AM $\rightarrow$ weak circulation
- AM loss by wind: need to transport AM to surface $\rightarrow$ strong circulation

Tachocline circulation in vicinity of conv. zone Spiegel & JPZ (1992)

Mixing processes in radiation zones

Turbulence caused by horizontal shear $\Omega(\theta)$

(barotropic instability)

Properties:

- instability acts to suppress its cause, i.e. rotation in latitude $\Omega(\theta)$
- turbulent transport is anisotropic (due to stratification): $D_h >> D_v$

Main weakness: no firm prescription for D_h

Maeder 2003
Mathis, Palacios & JPZ 2004

→ 2 important properties:

- erodes stabilizing effect of stratification

$$D_v = \nu_v = \frac{8}{5} \frac{Ri_{\rm crit}(r{\rm d}\Omega/{\rm d}r)^2}{N_T^2/(K + D_h) + N_\mu^2/D_h}$$

Talon & JPZ 1997

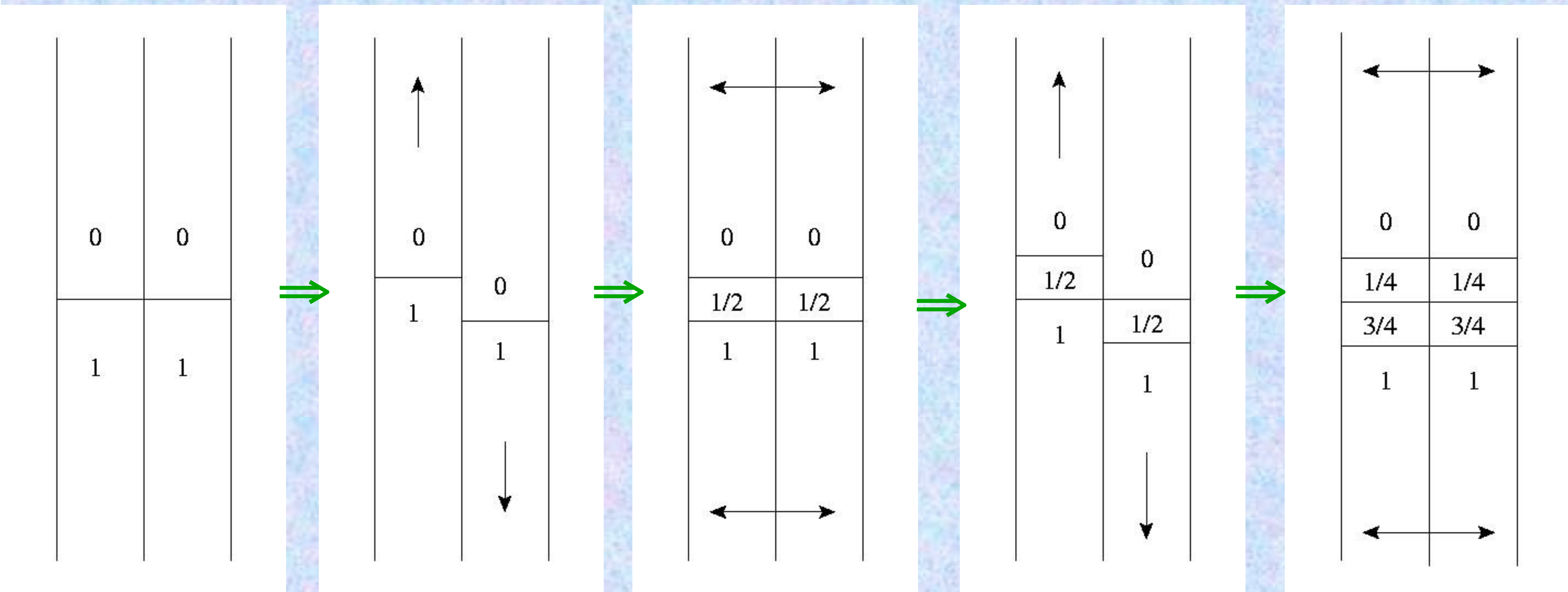
- changes advection of chemicals into vertical diffusion

$$D_{\rm eff} = \frac{1}{30} \frac{(rU)^2}{D_h}$$

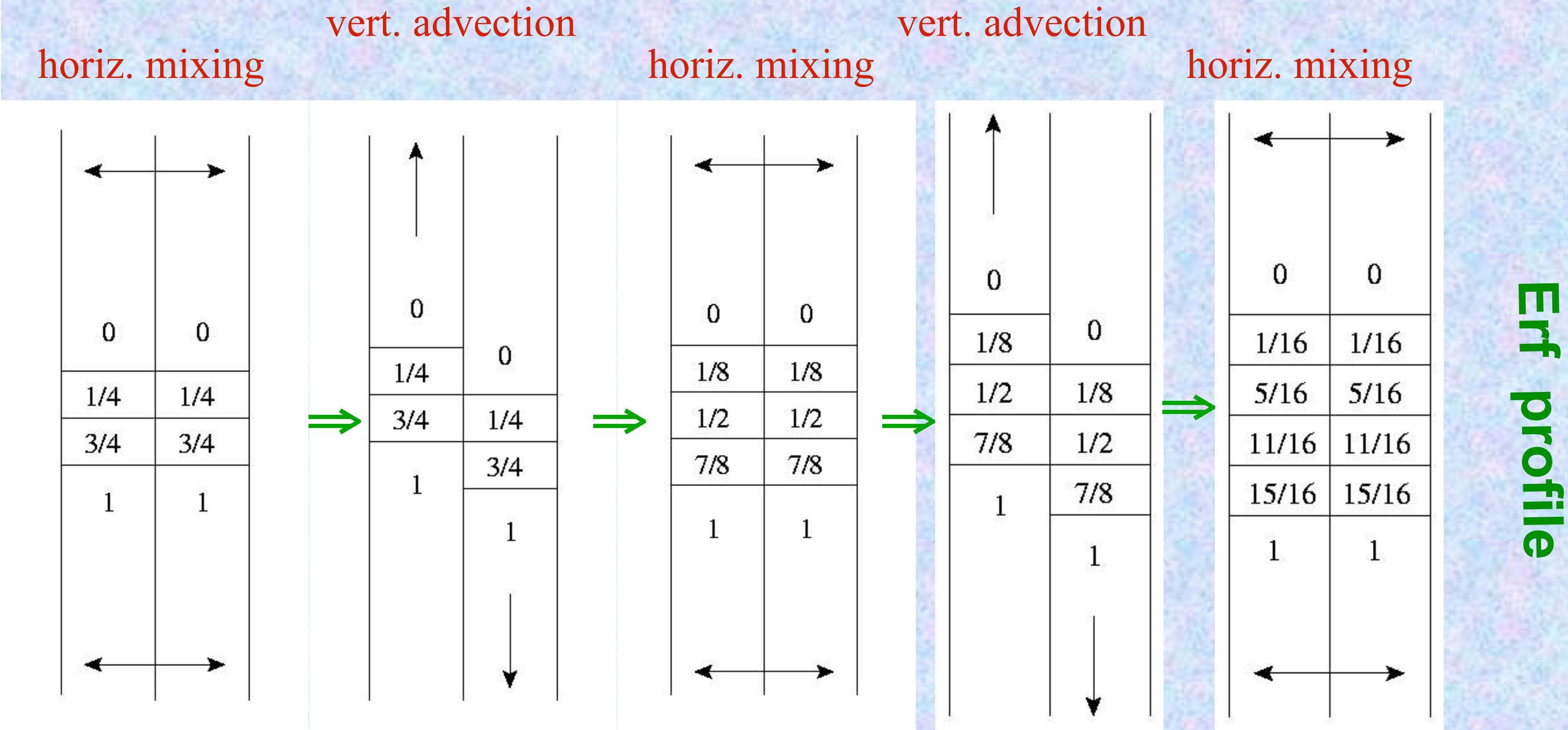
Chaboyer & JPZ 1992

A cartoon: Turbulent erosion of advective transport

initial state vertical advection horizontal mixing vertical advection horizontal mixing



Turbulent erosion of advective transport (cont.)



advective transport of chemicals ⇒ vertical diffusion

transport of AM remains an advective process

Mixing processes in radiation zones

Turbulence caused by vertical shear $\Omega(\mathbf{r})$
(baroclinic instability)

- if maximum of vorticity (inflexion point) : linear instability
- if no maximum of vorticity : finite amplitude instability
- stabilizing effect of stratification reduced by radiative diffusion

turbulence if

$$Ri_c \left(\frac{dV_{\rm hor}}{dr} \right)^2 > N^2 \left(\frac{w\ell}{K} \right)$$

Richardson criterion

from which one deduces the turbulent diffusivity (if μ = cst)

$$D_v = w\ell = Ri_c K \frac{\Omega^2}{N^2} \left(\frac{d \ln \Omega}{d \ln r} \right)^2$$

Townsend 1959
Dudis 1974; JPZ 1974
Lignières et al. 1999

K thermal diffusion; ν viscosity; N buoyancy frequency

Rotational mixing - the observational test

Assumption: the processes (i.e. circulation and turbulence)
that cause the mixing of chemical elements
are also responsible for the transport of angular momentum
JPZ (1992), Maeder & Z (1998)

- quite successful with early-type stars (fast rotators)
  Talon et al. 1997; Maeder & Meynet 2000; Talon & Charbonnel 1999
- for late-type stars (which are spun down by wind) predicts
  - fast rotating core ≠ helioseismology
  - strong destruction of Be in Sun
    not observed
  - mixing correlated with loss of angular momentum
    ≠ Li in tidally locked binaries
    ≠ little dispersion in the Spite plateau

⇒ Another, more powerful process is responsible
for the transport of angular momentum

2 types of rotational mixing

In any case, circulation and turbulence
are responsible for the mixing of chemical elements

Rotational mixing of type I :
angular momentum is carried also by
circulation and turbulence

Rotational mixing of type II :
another process operates
for the transport of angular momentum;
has indirect impact on mixing, by shaping $\Omega(\mathbf{r})$

- magnetic field ?
- internal gravity waves ?

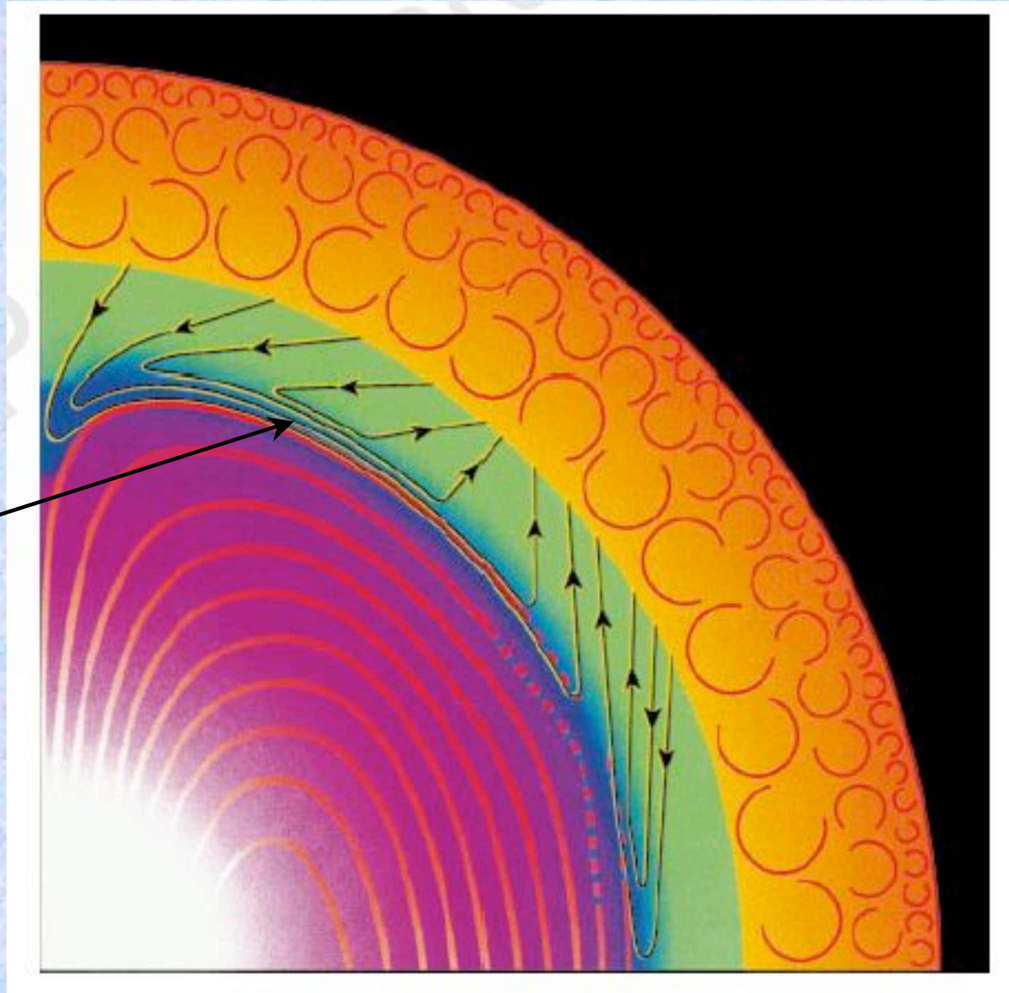
Role of a fossil magnetic field

Does it prevent the spread of tachocline?
Does it enforce uniform rotation?

convection zone

tachocline
void of magnetic field

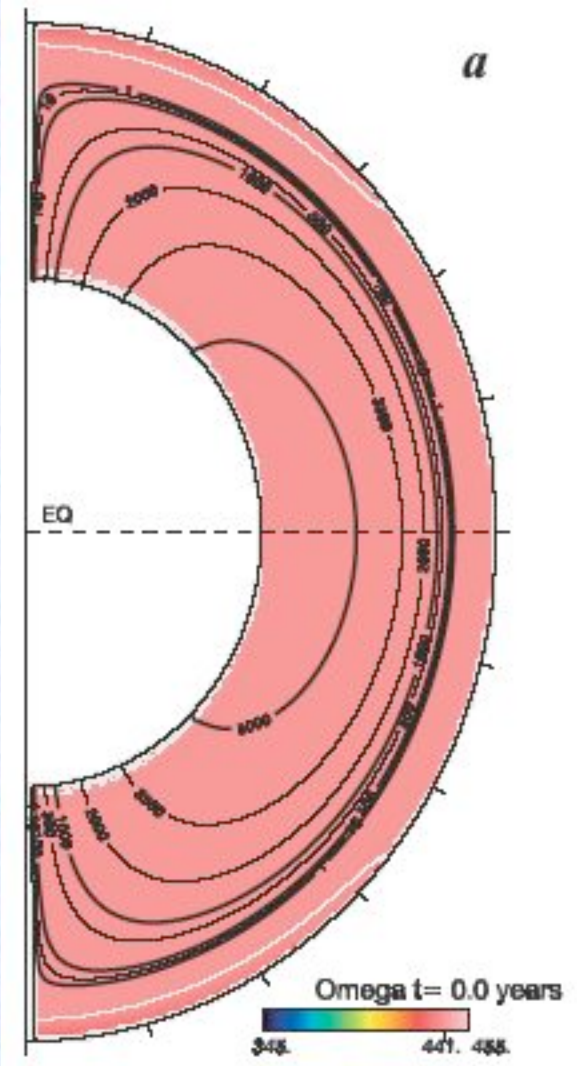
magnetopause



YES, according to
Gough & McIntyre 1998

Role of a fossil magnetic field

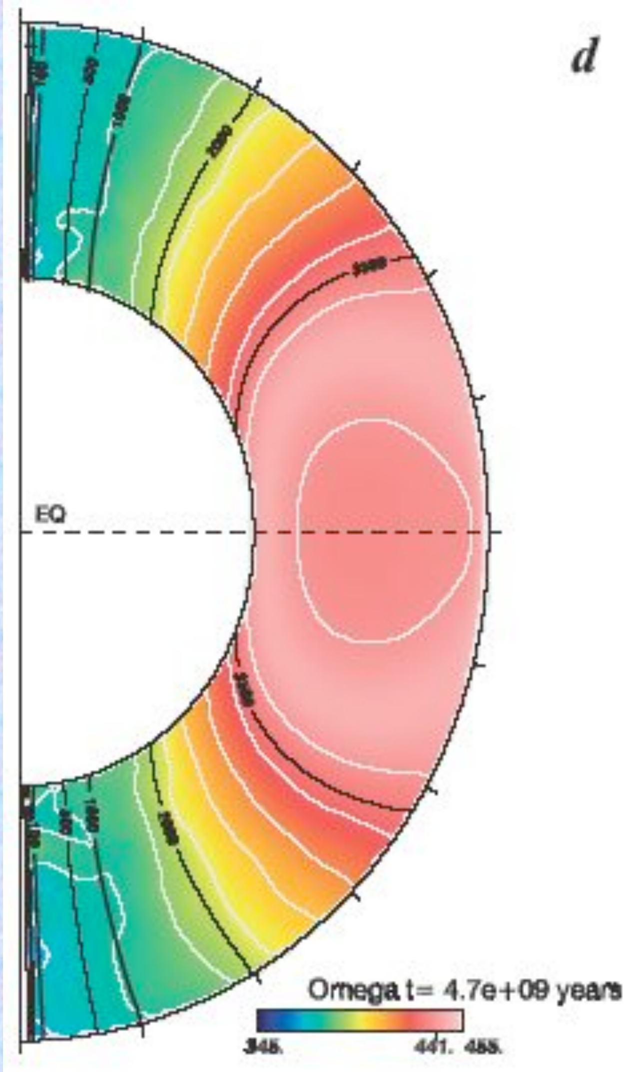
Can such a field rigidify the rotation in the radiation zone ? NO



3D time-dependent
solutions
ASH code

← initially
deeply buried
poloidal field

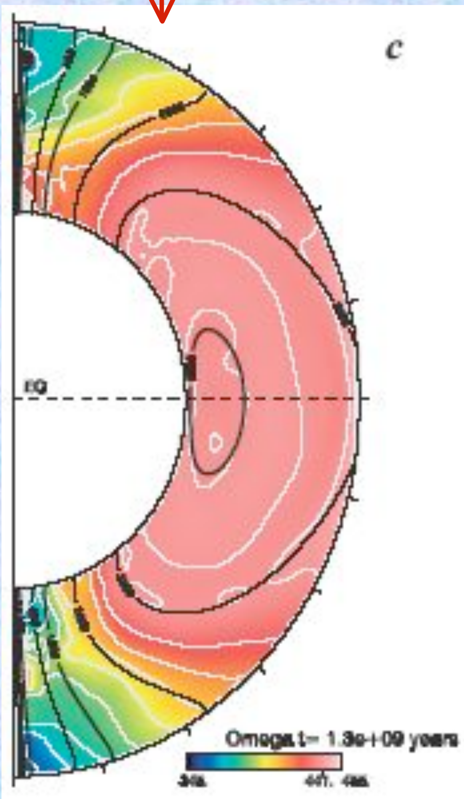
diffuses into CZ,
imprints diff. rot.
on RZ →
(Ferraro law)

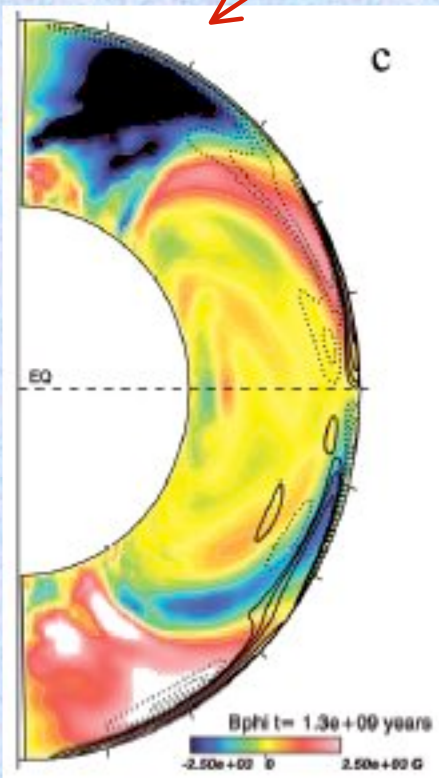


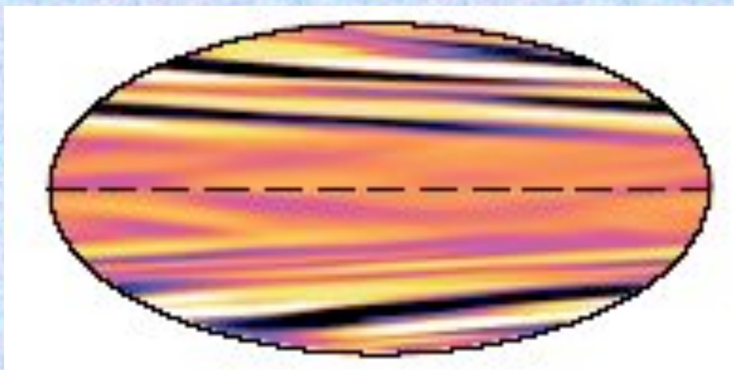
Brun & Zahn 2006

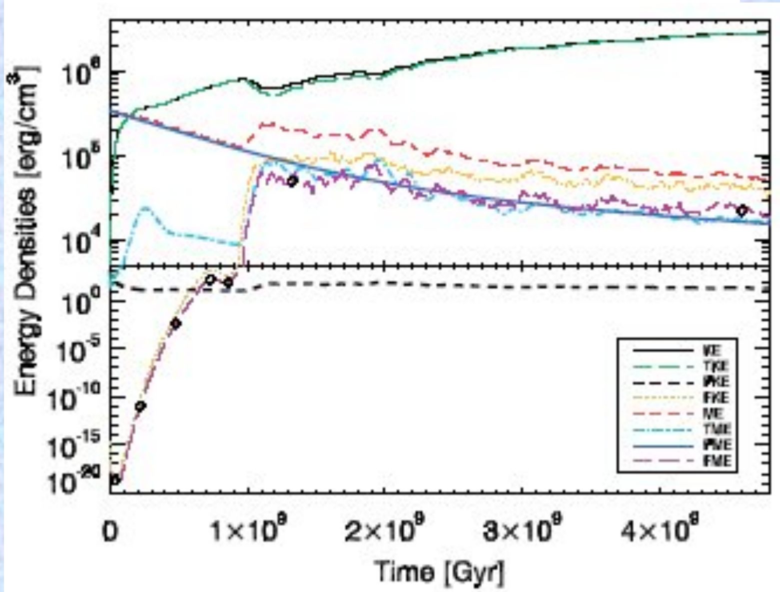
MHD instabilities Just an example - not in the Sun !

poloidal field + diff. rotation → toroidal field → instabilities









Pitts & Tayler 1986, Spruit 1999, Brun & Zahn 2006

Angular momentum transport by waves

Schatzman 1993, Zahn et al 1997, Talon et al 2002, Talon & Charbonnel 2005

Internal gravity waves and gravito-inertial waves
are emitted at the base of the convection zone

They transport angular momentum, which they deposit
where they are damped through thermal diffusion

- if prograde and retrograde waves are equally excited
and if there is no differential rotation

$\Rightarrow$ no net momentum deposition

- if there is differential rotation, prograde and retrograde waves
deposit their momentum at different locations

$\Rightarrow$ waves increase the local differential rotation,
until the shear becomes unstable $\Rightarrow$ turbulence

- high l waves are damped very close to the CZ

- low l, low frequency waves are damped in deep interior

Rotational mixing type II in solar type stars : the observational test

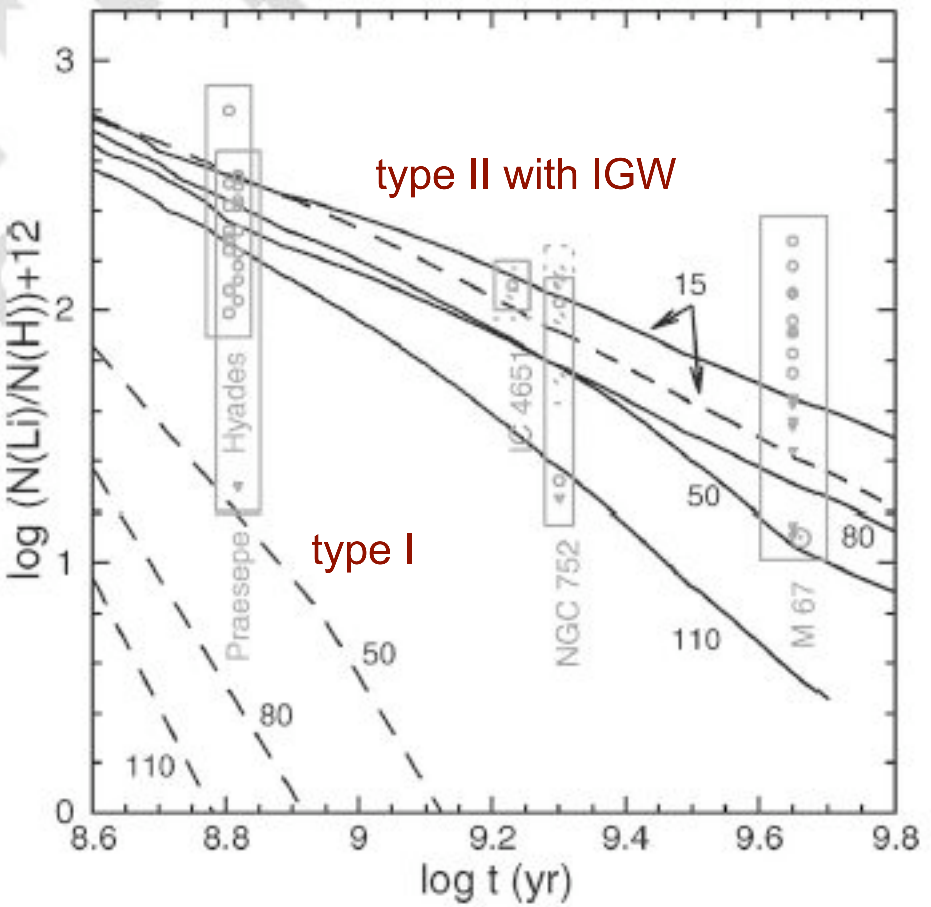
Circulation and turbulence are responsible
for the mixing of chemical elements

Angular momentum is transported
by internal gravity waves

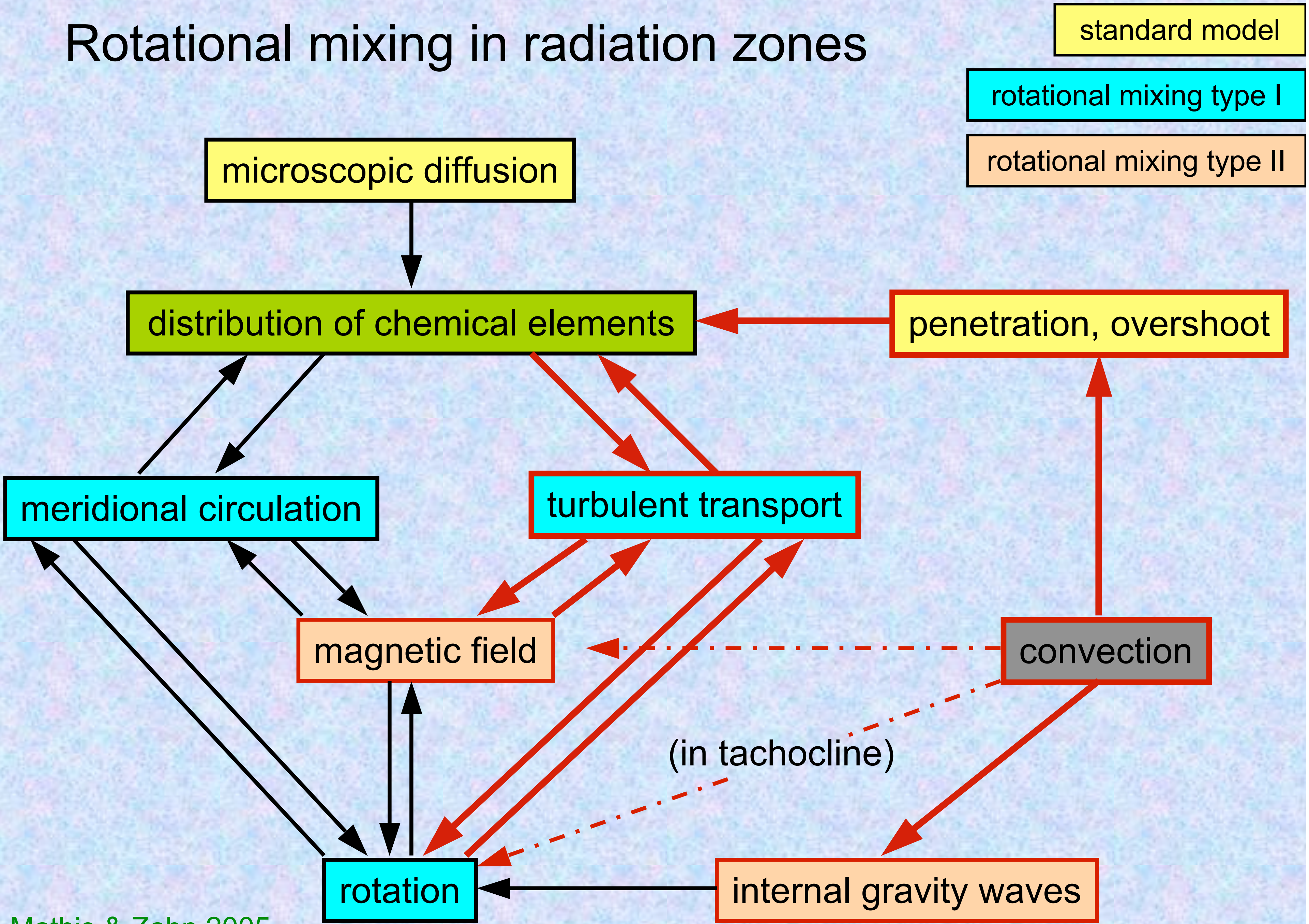
- IGW are able to extract AM
  from solar interior and
  to render the rotation uniform

- the right amount of Li is depleted
  in Pop I and Pop II stars

Charbonnel & Talon 2005



Rotational mixing in radiation zones



Mathis & Zahn 2005

Weakest points of present models with rotational mixing

- Parametrization of the turbulence caused by differential rotation

- Which physical process prevents the spread of the tachocline ?

- Power spectrum for IGW emitted at base of convection zone

- Particle transport by IGW ?

- Role of instabilities due to magnetic field ?