Turbulence in the convection-radiation transition layer of solar-type stars

The effects on oscillation frequencies

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Outline

- 3D Large Eddy Simulations (LES)

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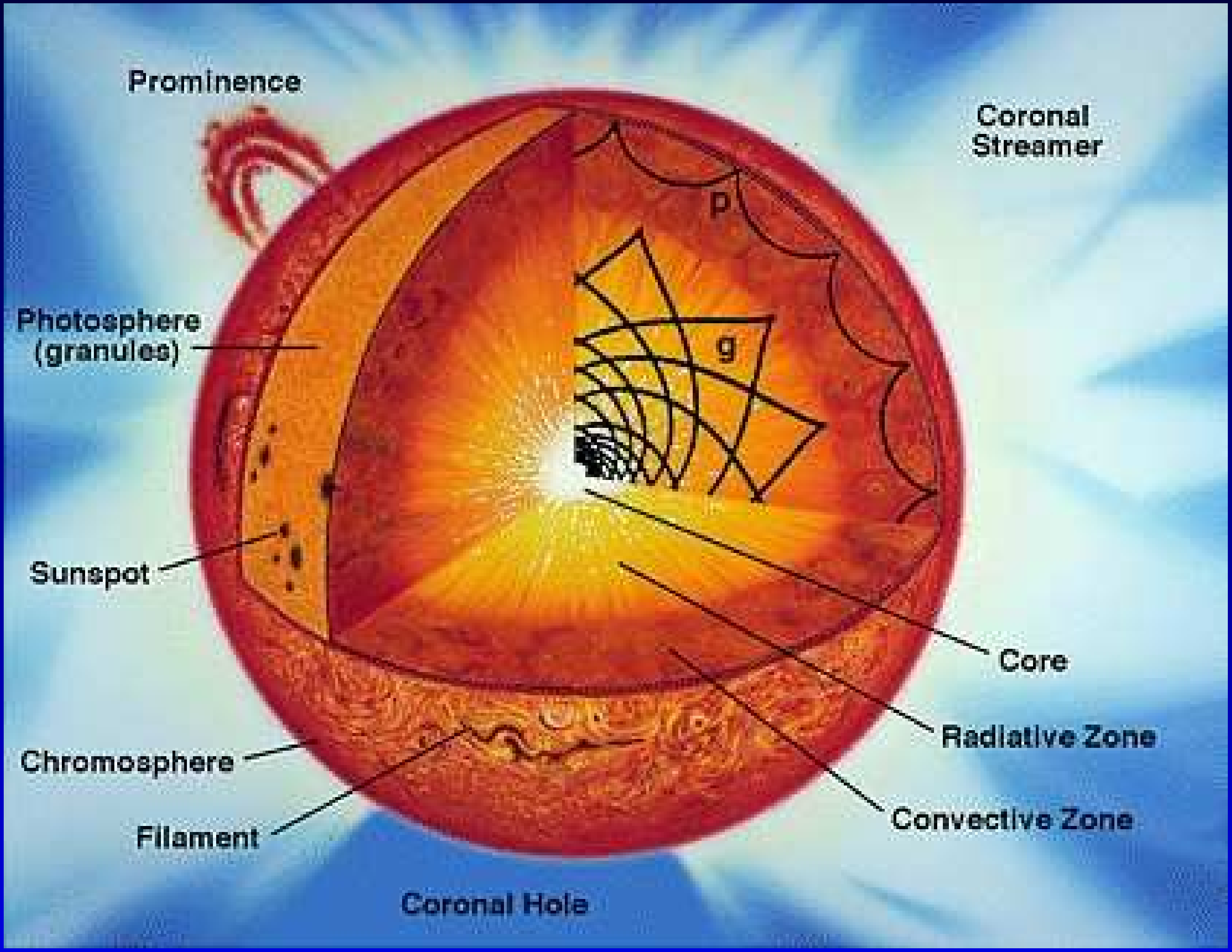
- Comparison of LES with MLT
- Effects on frequencies:
  Turbulence vs. Gravitational Settling
- Conclusions

The Sun



Source: J.W. Leibacher, National Solar Observatory

SAL in Solar Modeling

The Sun

- outer layers ($r > 0.98\,R_\odot$) cannot be probed by inversion (lack of high degree modes!)
- super-adiabatic layer (SAL) located at $0.9998\,R_\odot$
- effects of turbulent convection dominant at SAL peak

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Modeling the convection-radiation layer

- extremely high Re
- MLT not adequate
- $\Rightarrow$ one approach: LES

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$$\begin{aligned}\frac{\partial \rho}{\partial t} &= -\nabla \cdot \rho \mathbf{v} \\ \frac{\partial (\rho \mathbf{v})}{\partial t} &= -\nabla \cdot (\rho \mathbf{v})\mathbf{v} - \nabla P + \nabla \cdot \mathbf{\Sigma} + \rho \mathbf{g}\end{aligned}$$

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\Sigma_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2\mu}{3} (\nabla \cdot \mathbf{v})\, \delta_{ij}
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- Large Eddy Simulation (LES), Smagorinsky (1963) viscosity:

$$
\mu = \rho (c_\mu \Delta)^2 (2\boldsymbol{\sigma} : \boldsymbol{\sigma})^{1/2}
$$

Recent Improvements

- 3D Radiative Transfer: Multigrid Method

$$\nabla \cdot \left(\frac{1}{3\kappa\rho} \nabla J \right) - \kappa\rho J + \kappa\rho B = 0$$

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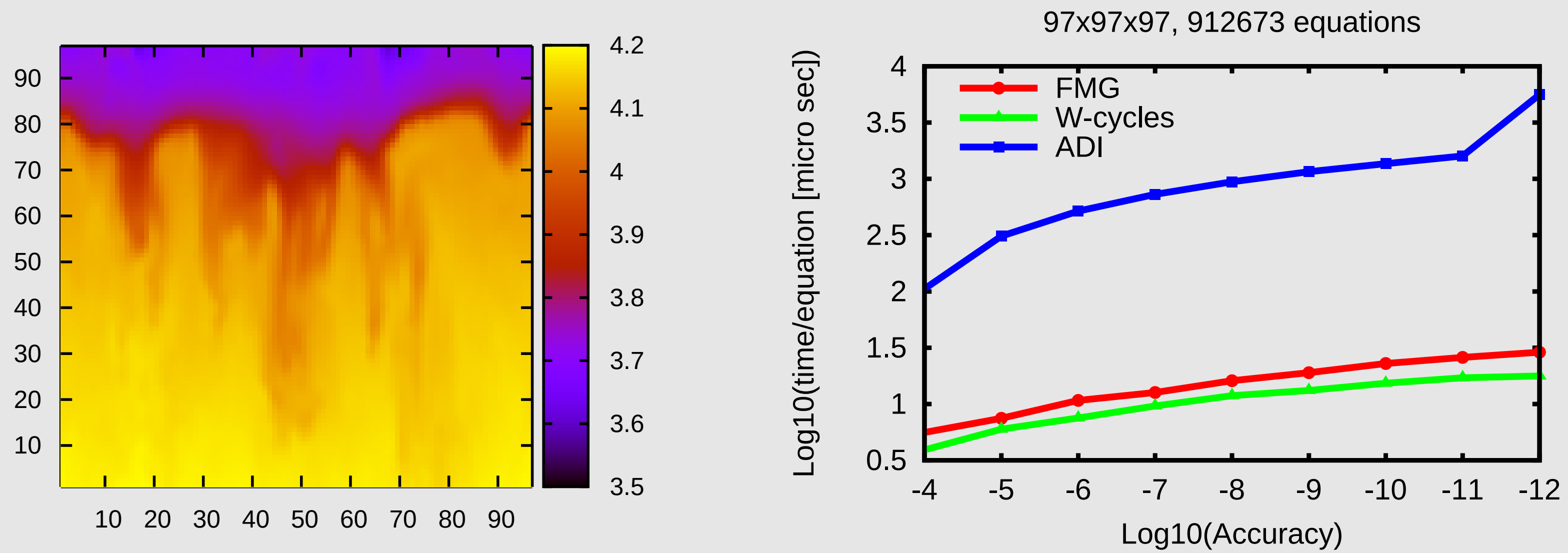

Straka & Demarque 2006, JQSRT, submitted

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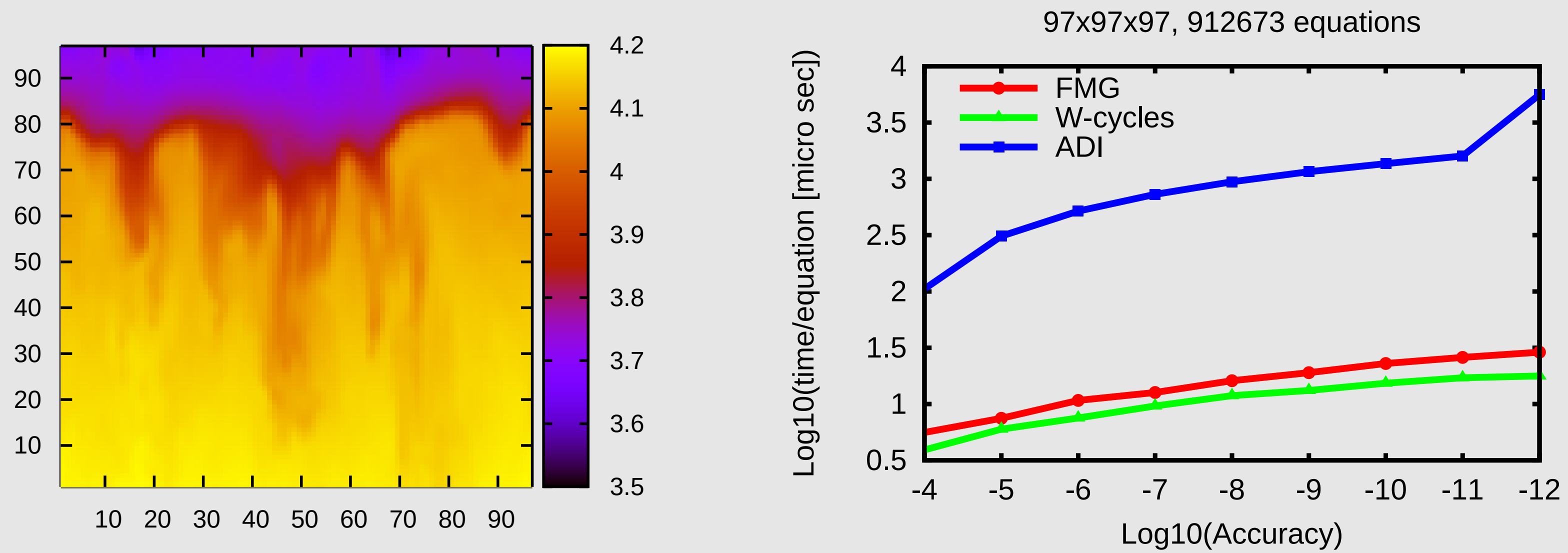

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- Simulation box above SAL peak: 2-3 H_P

Recent Improvements (continued)

- Treatment of Shocks:
  Generalized Richtmyer-Mortan scheme

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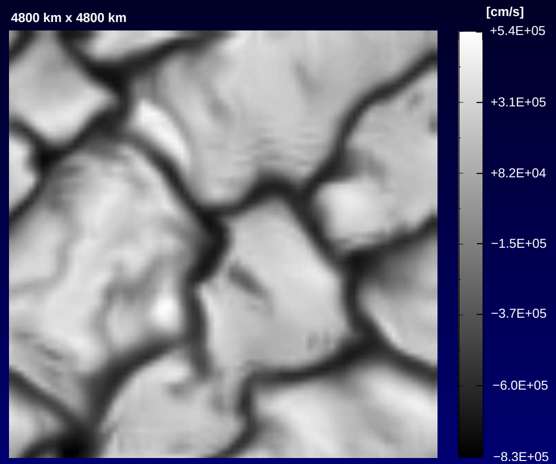
Currently working on:

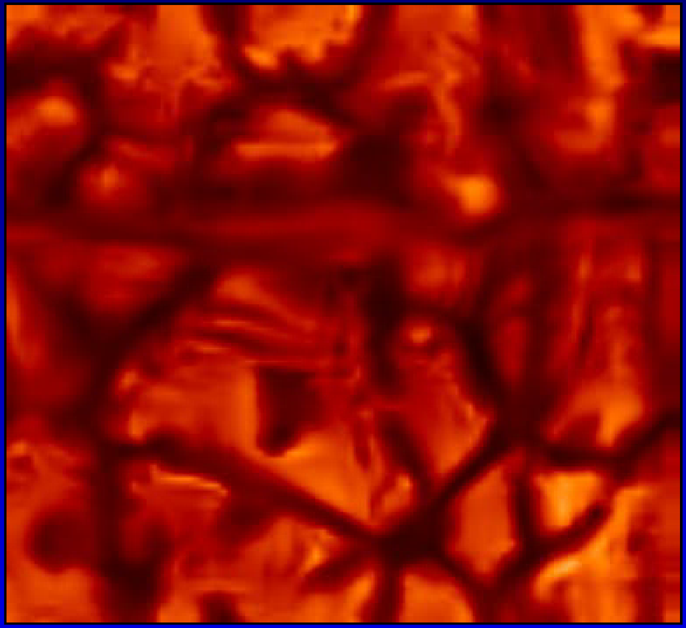
- Non-Gray radiative transfer
- Parallelization

LES of individual Stars

- Sun ....
  (Robinson et al. 2003)

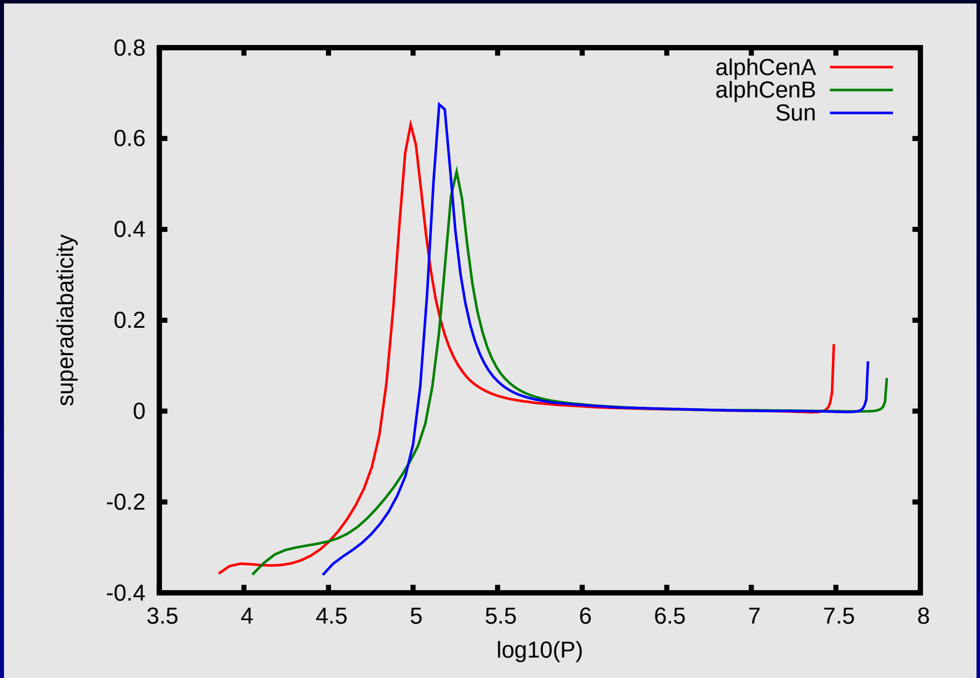


- η Bootis ....



- α Centauri A, B
- Asplund Sun

3D Simulation of α Cen: SAL



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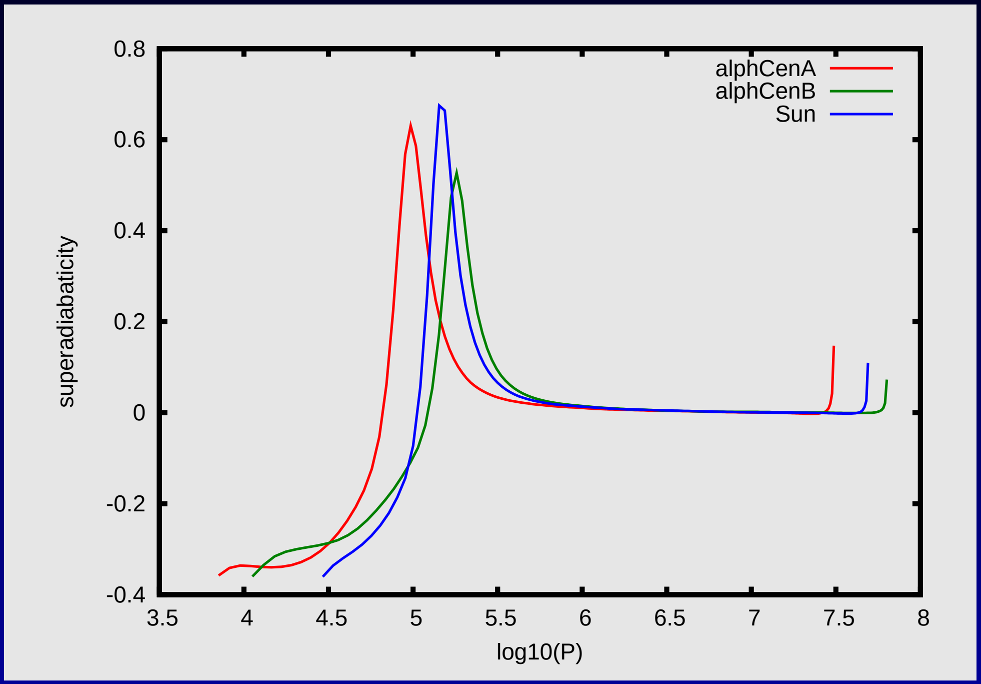

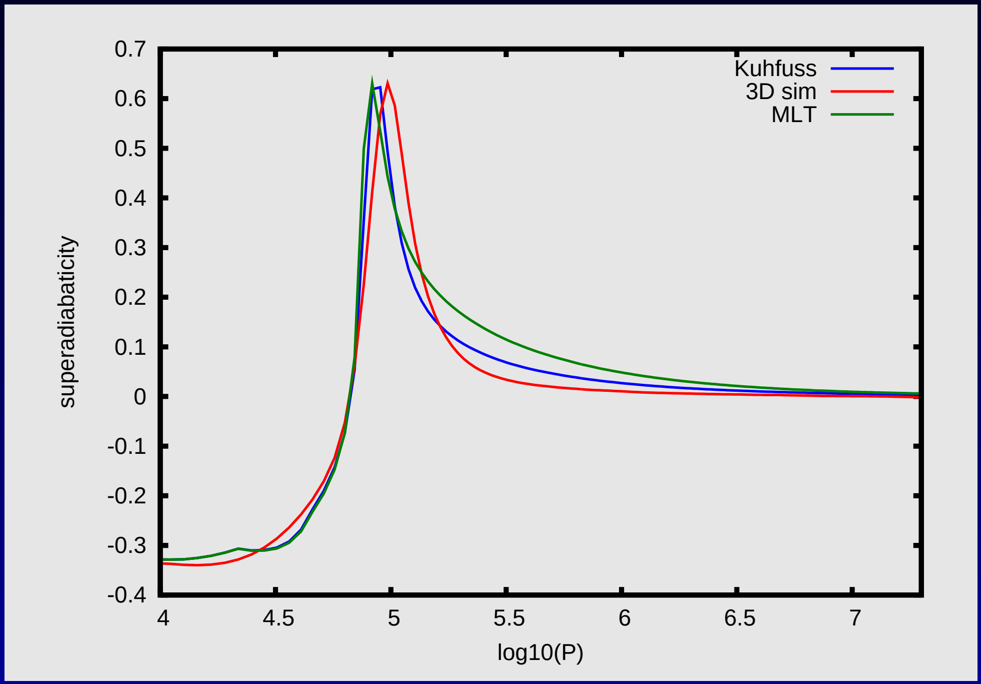

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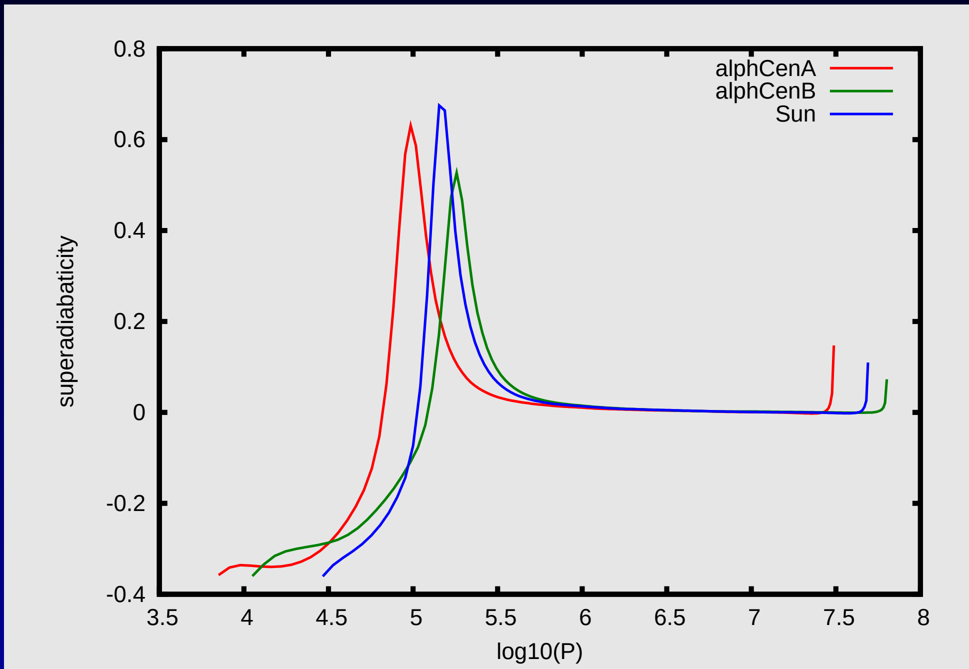

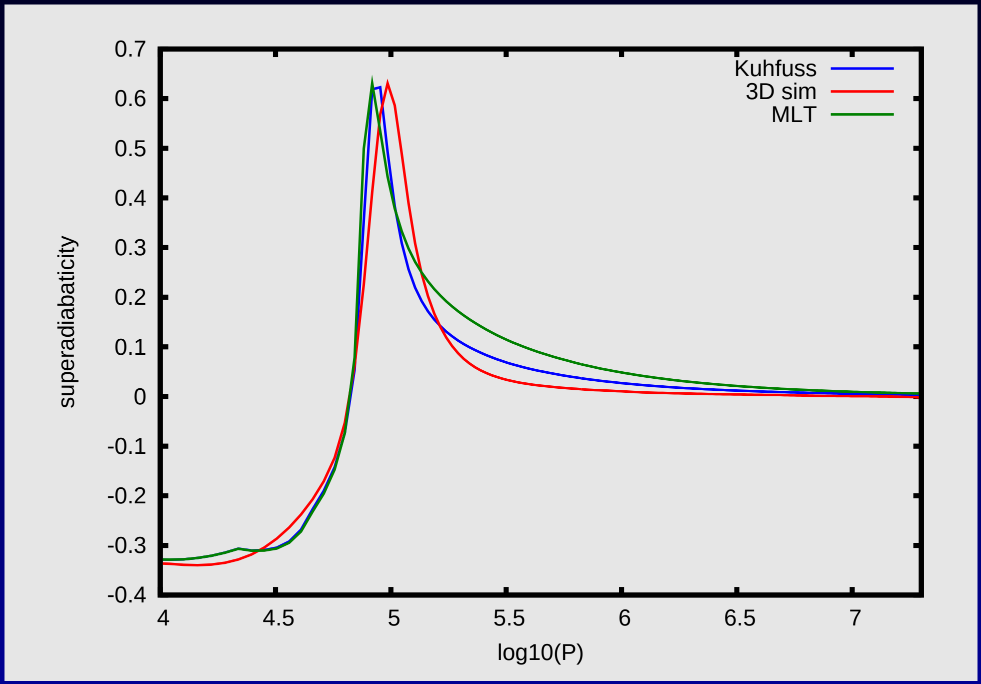

- α Cen A and B bracket the Sun
- MLT under-estimates adiabaticity!
- Non-local convection theories are better (but not perfect)

Inclusion in 1D Stellar Modeling

- Li et al. 2002

$$\chi = \frac{1}{2} v_{\mathrm{turb}}^2 \,, \quad v_{\mathrm{turb}} = \left(\overline{v_i^2} - \overline{v_i}^2 \right)^{1/2}$$

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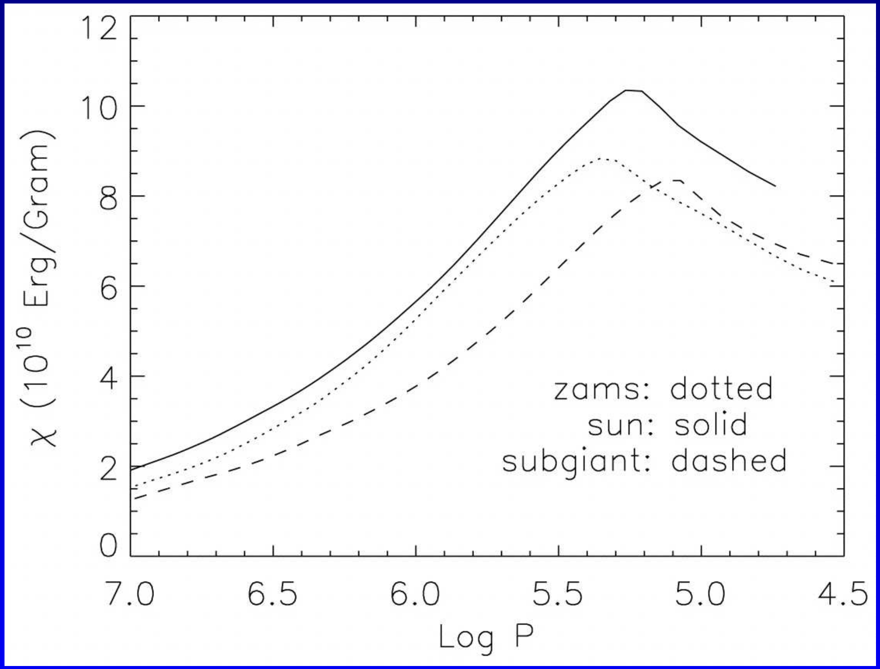

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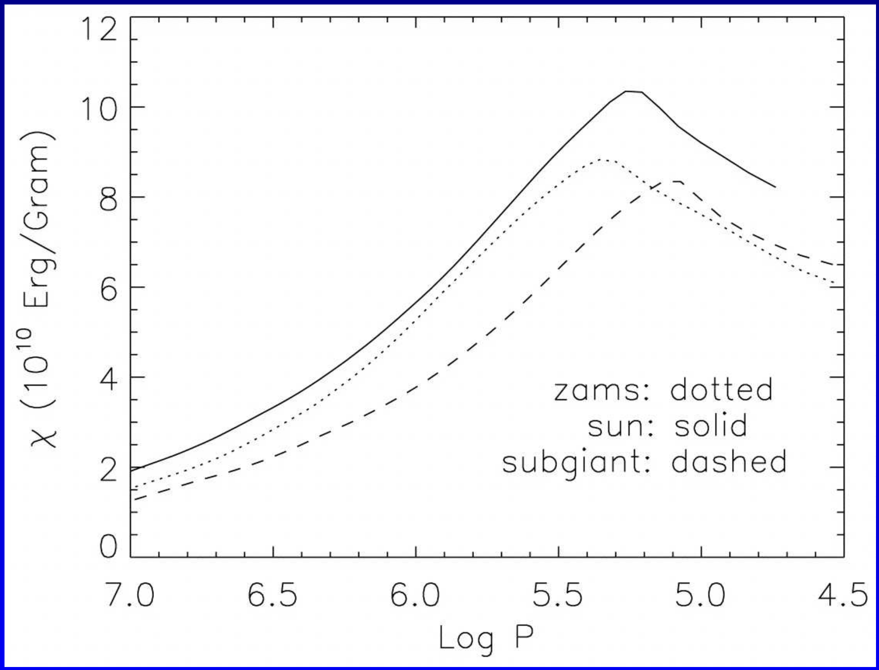
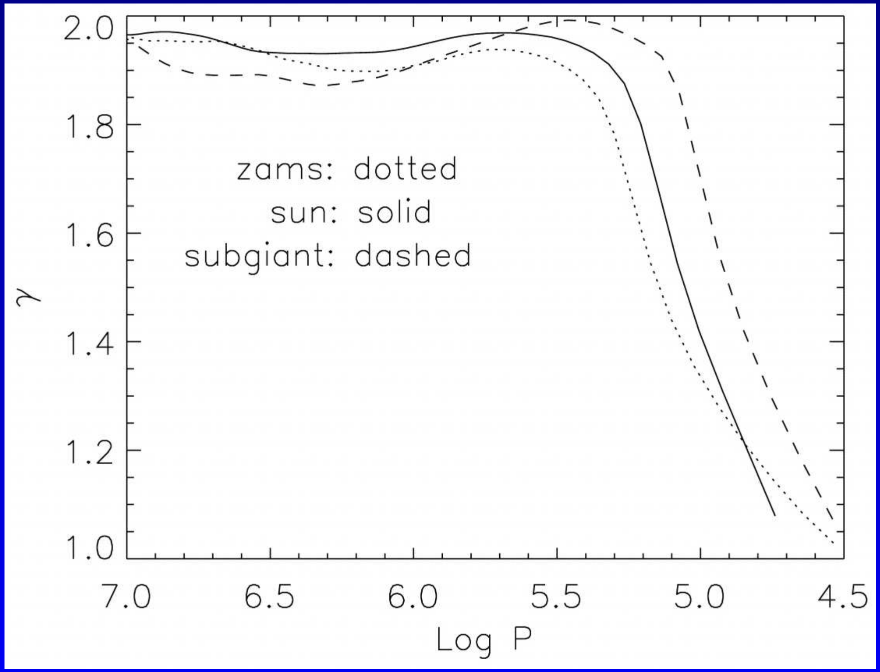
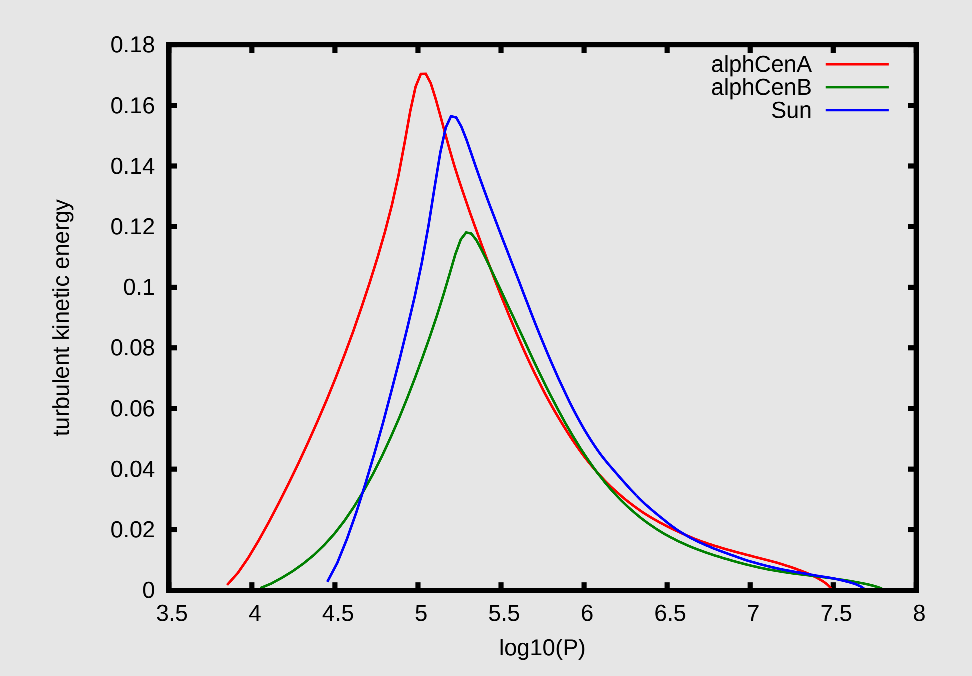




3D Simulation of α Cen

Turbulent Kinetic Energy



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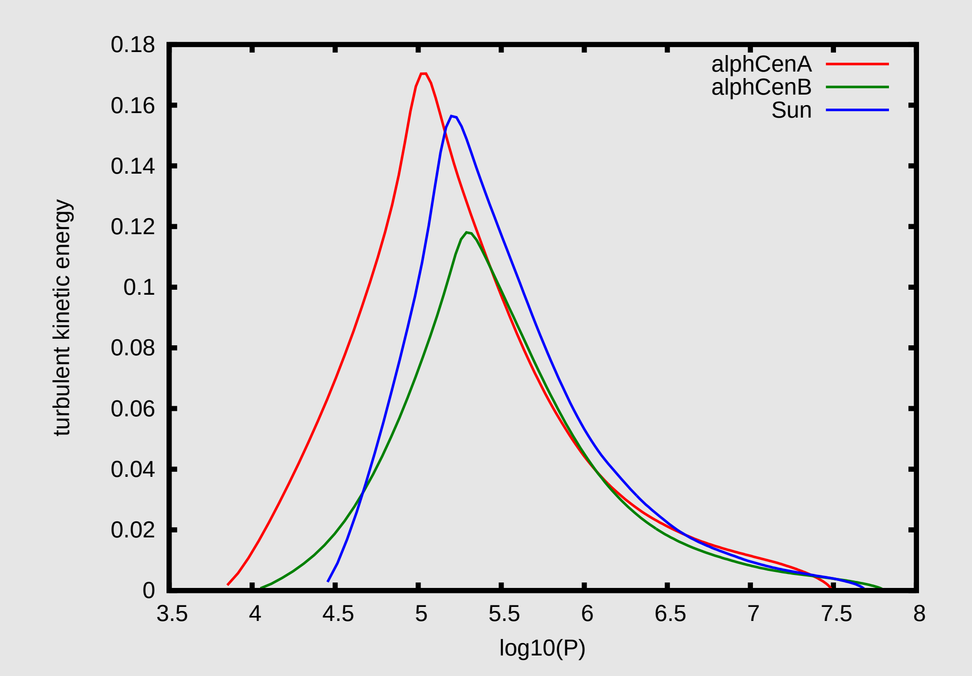
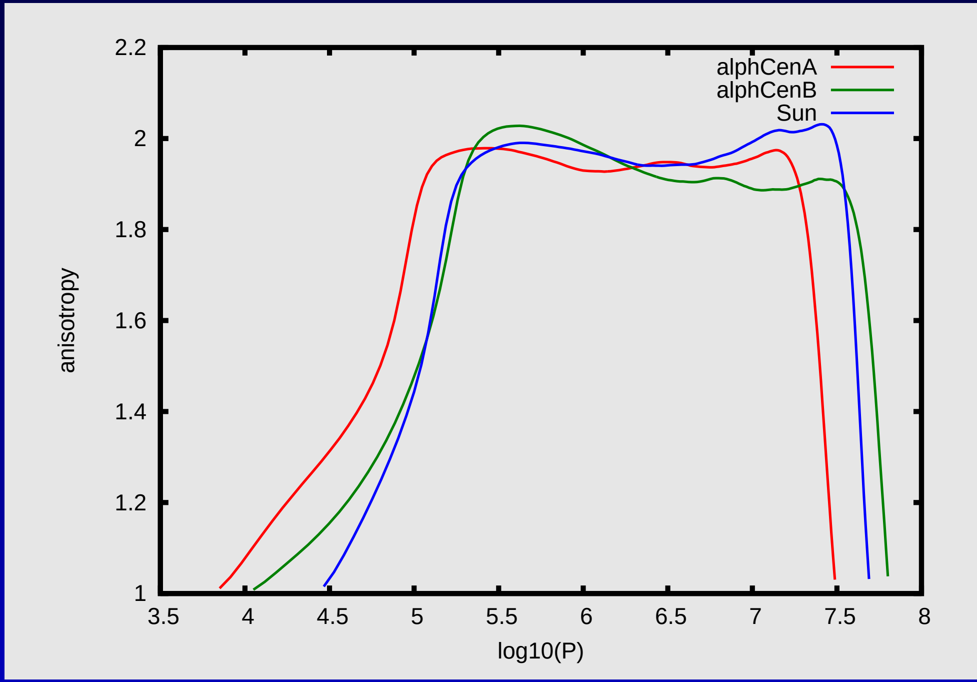
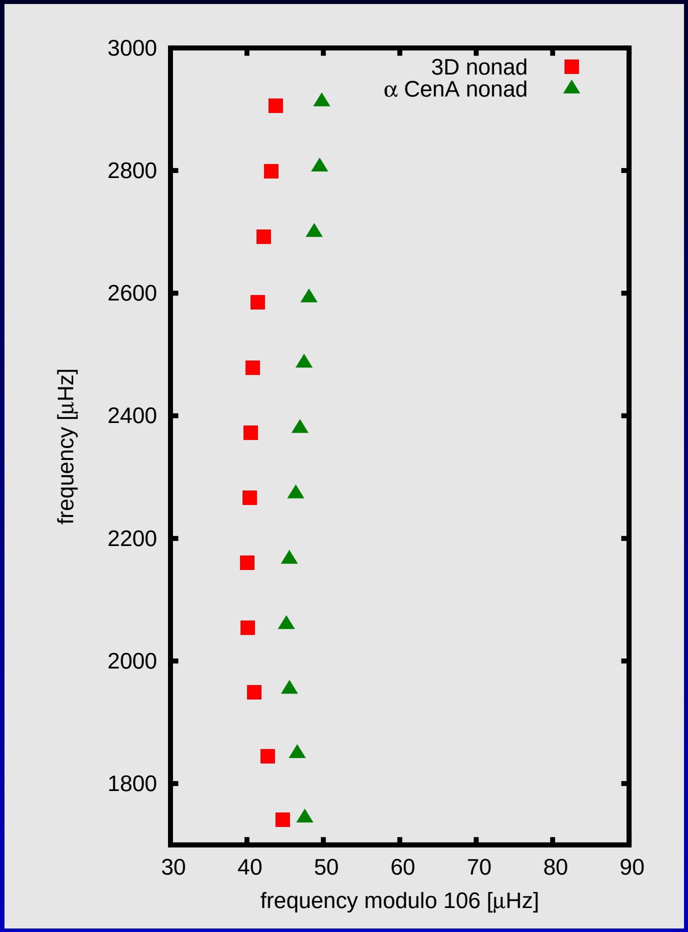


Anisotropy



p-mode Frequencies

α Cen A



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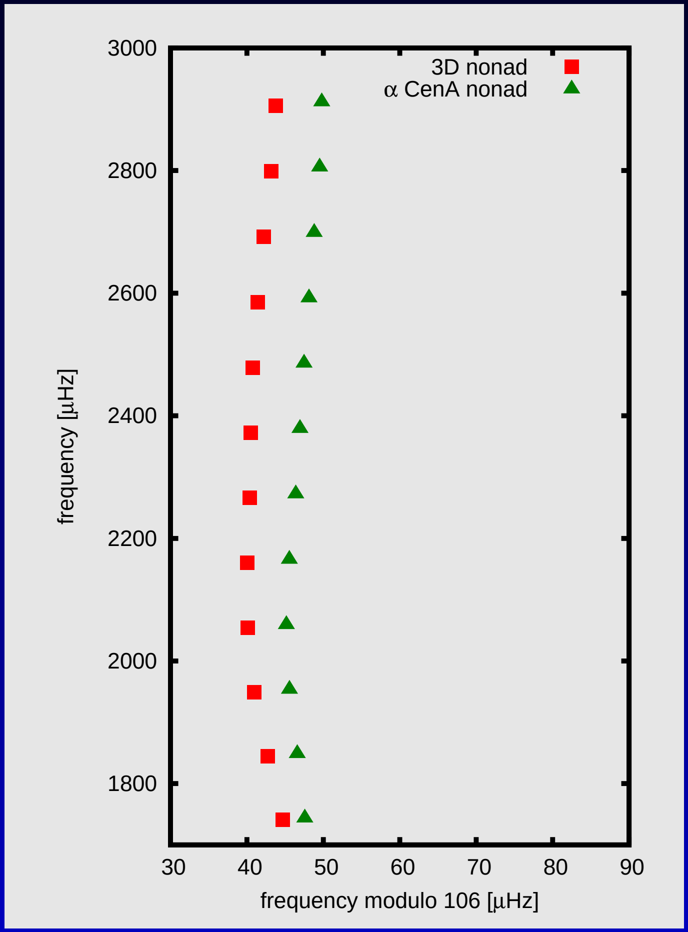
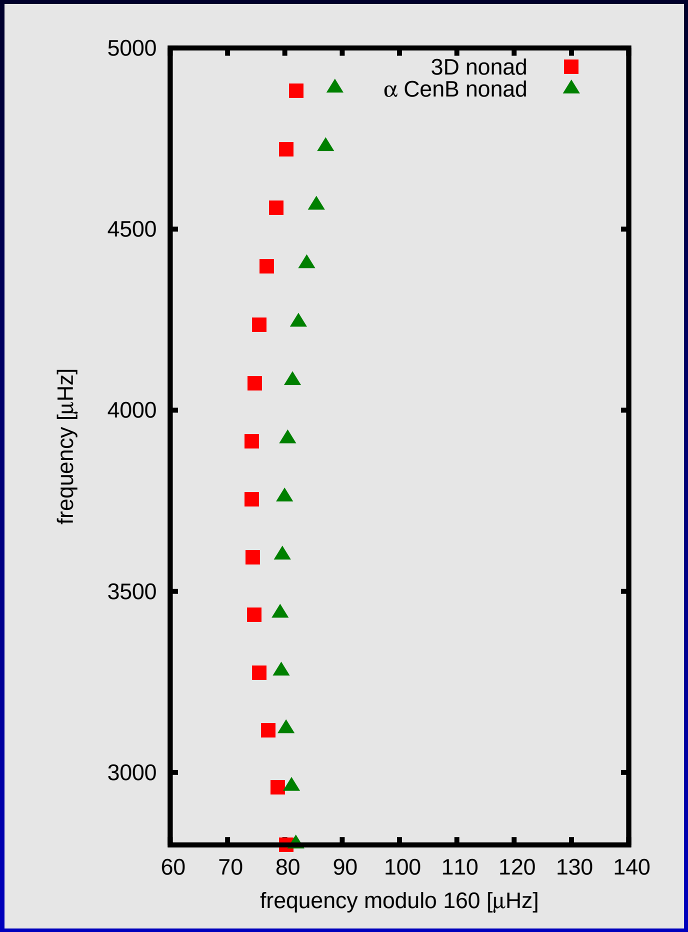


α Cen B



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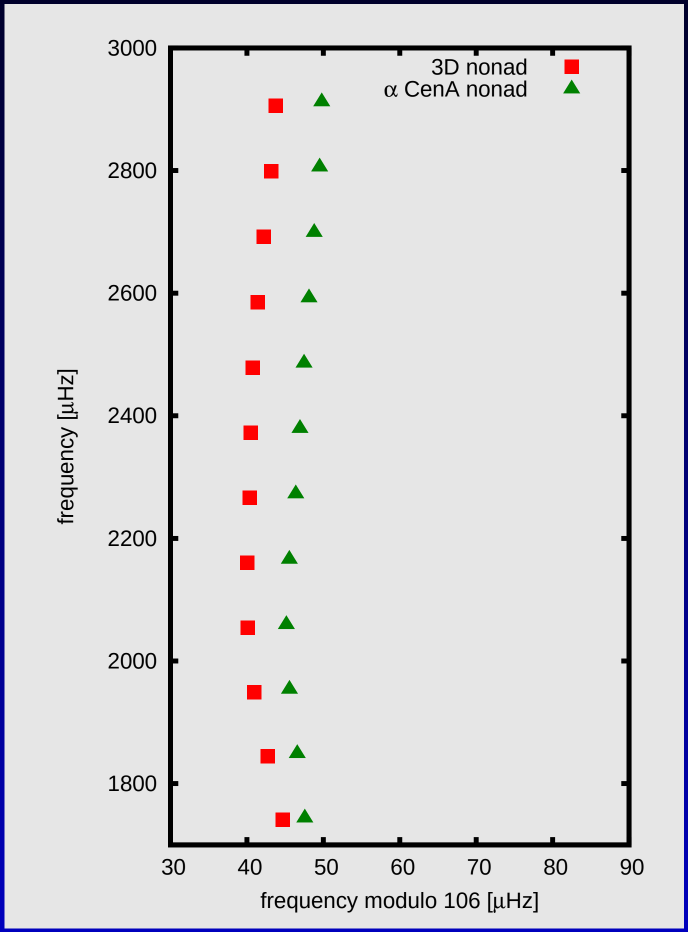
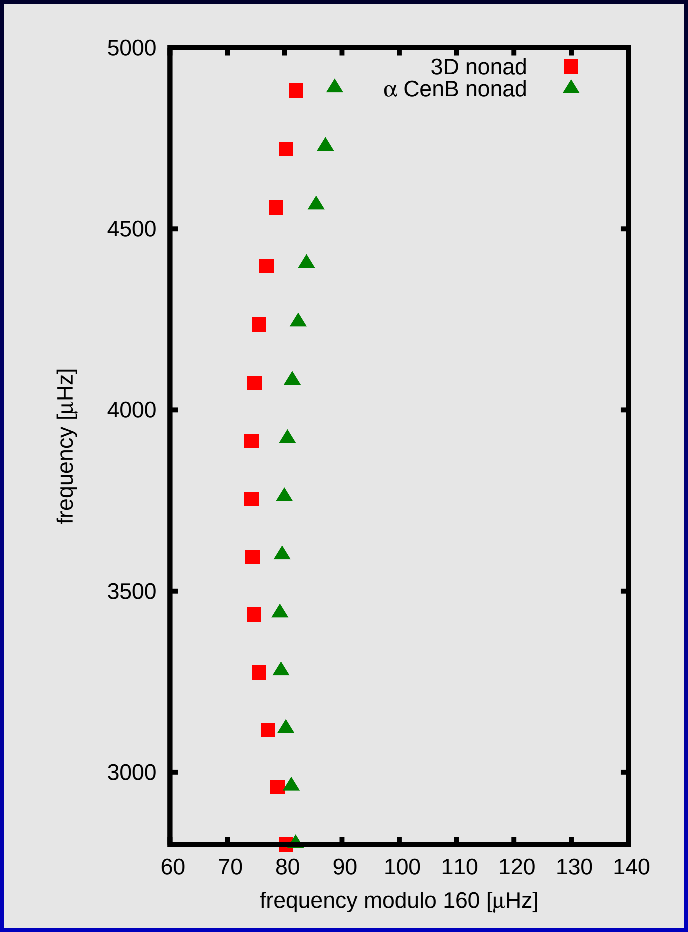


α Cen B



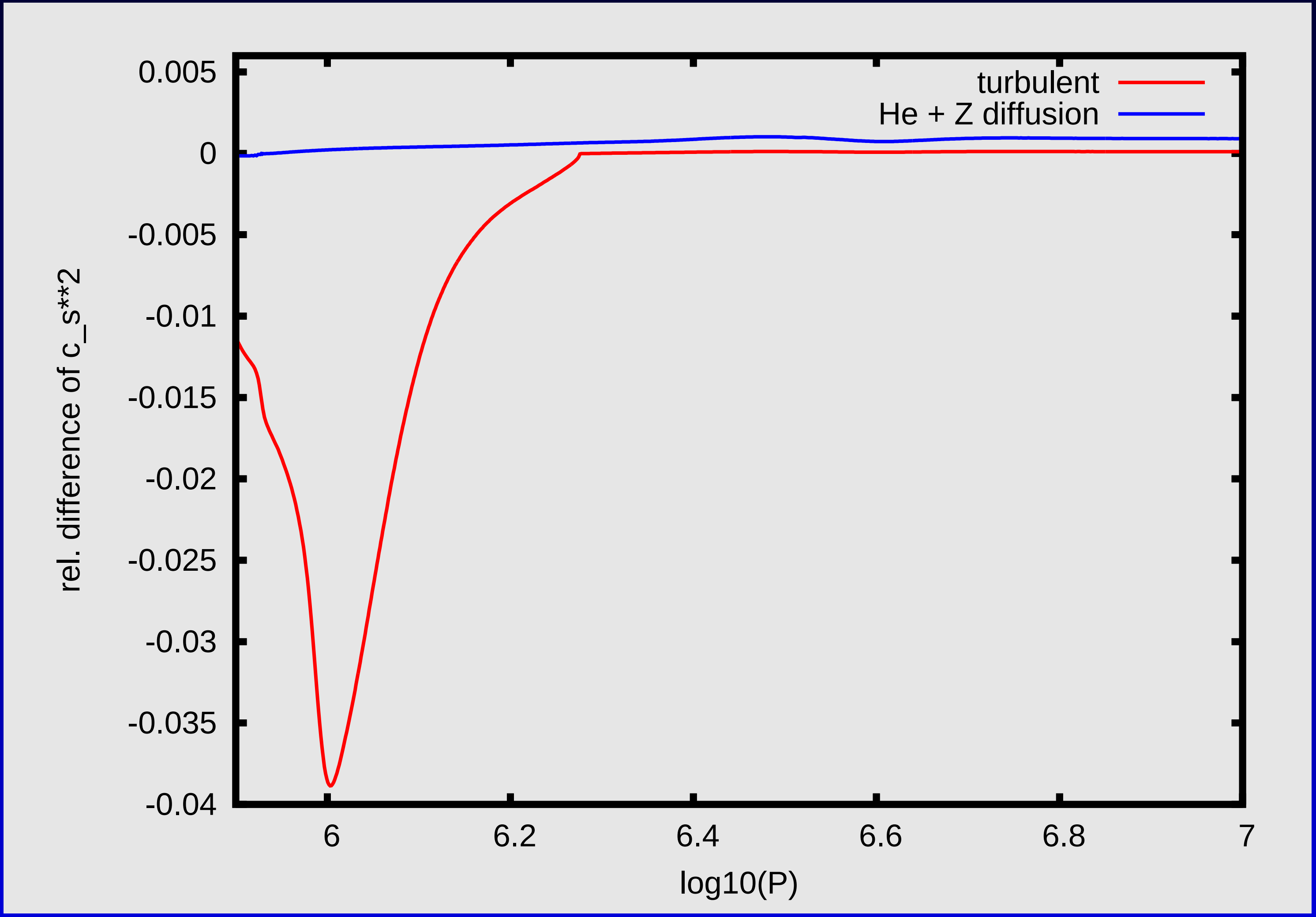
- frequency shift due to turbulence
- frequency shift in range of observed modes
- both A and B component affected

Comparison to Gravitational Settling

- standard model: 6.8 Gyrs, calibrated to observed L,R
- turbulence model: modified SAL due to LES, calibrated
- diffusion model: He and Z diffusion, calibrated

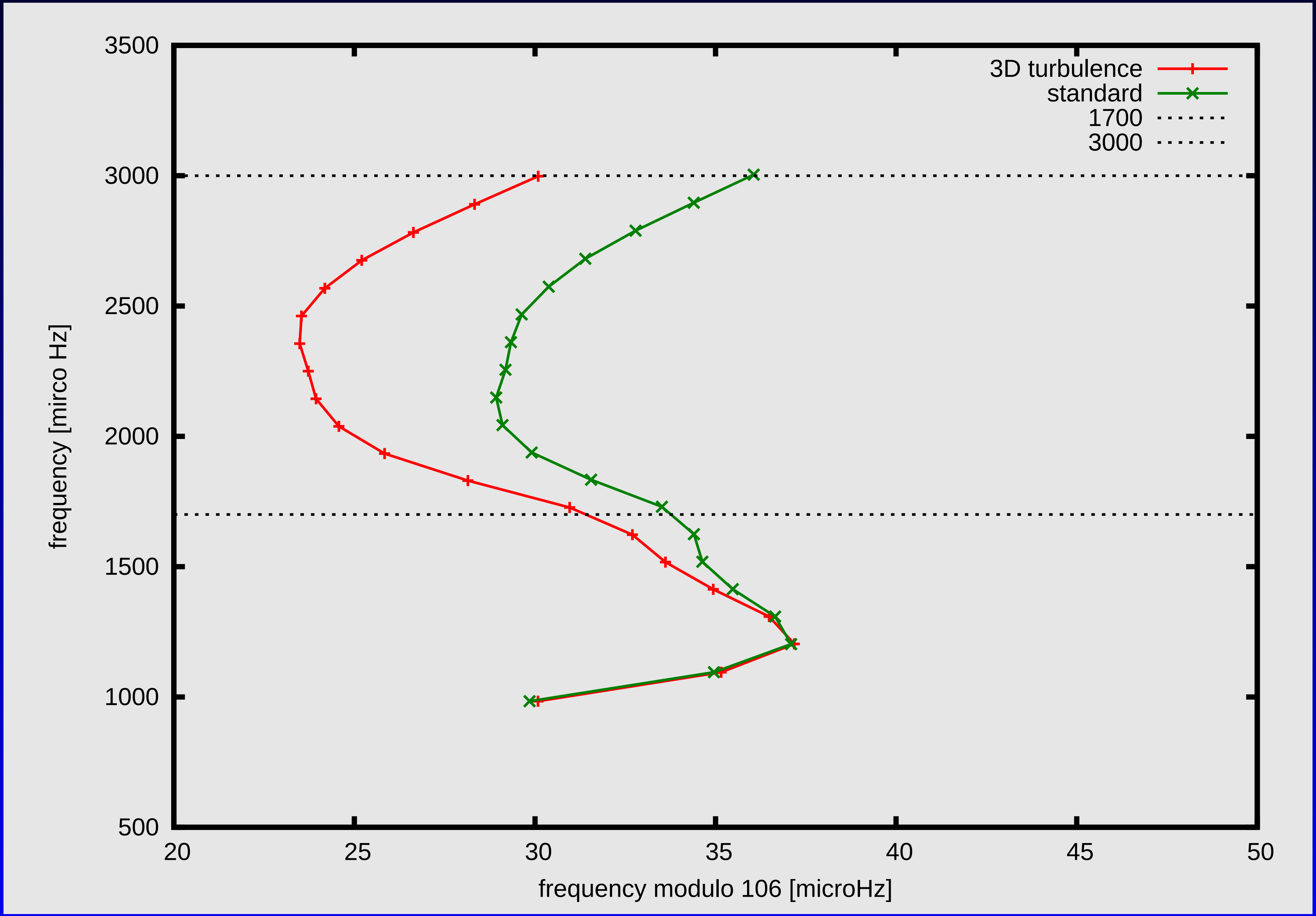
How do the two different effects compare?

Gravitational Settling



Gravitational Settling

Echelle Diagram α Cen A



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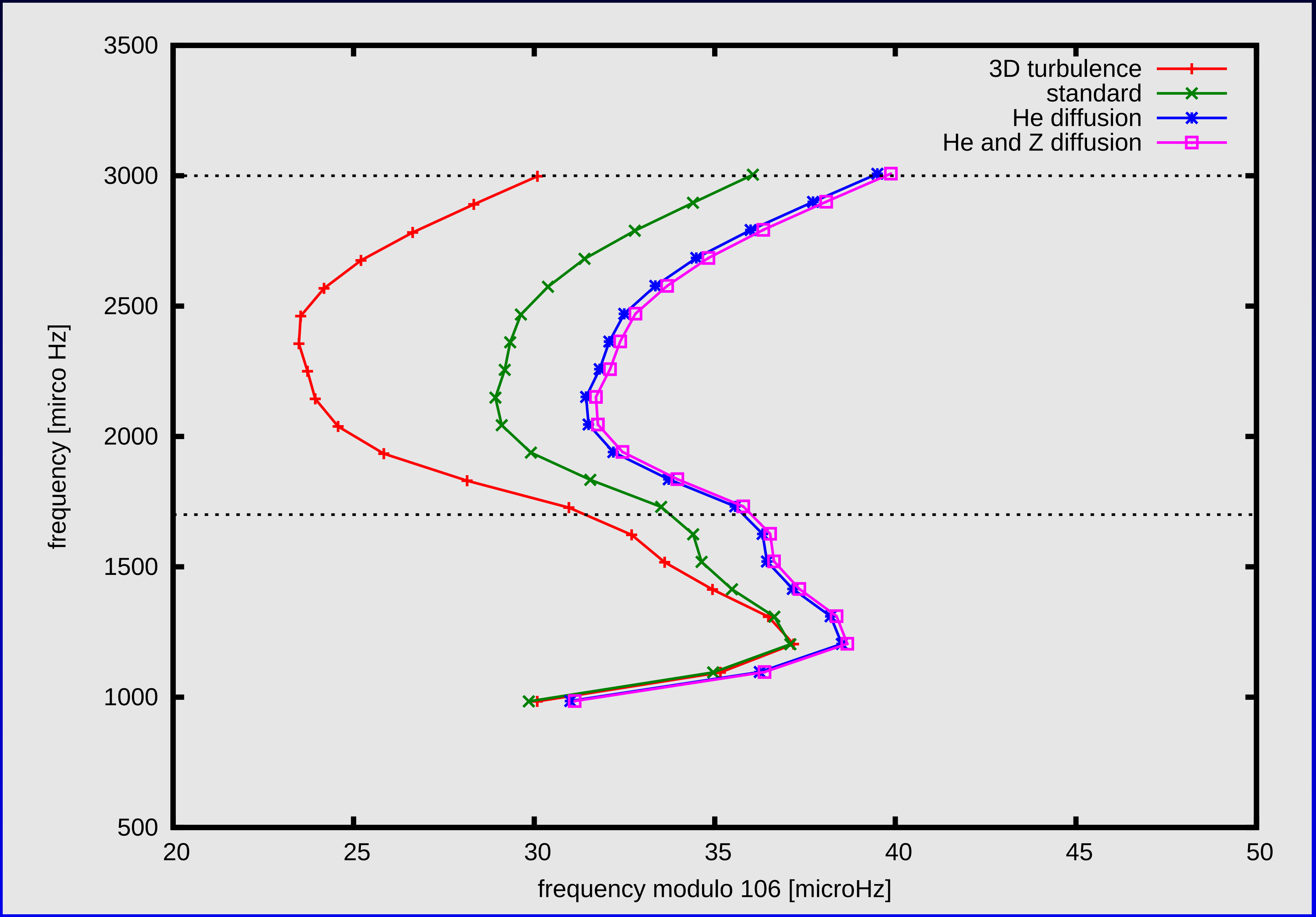

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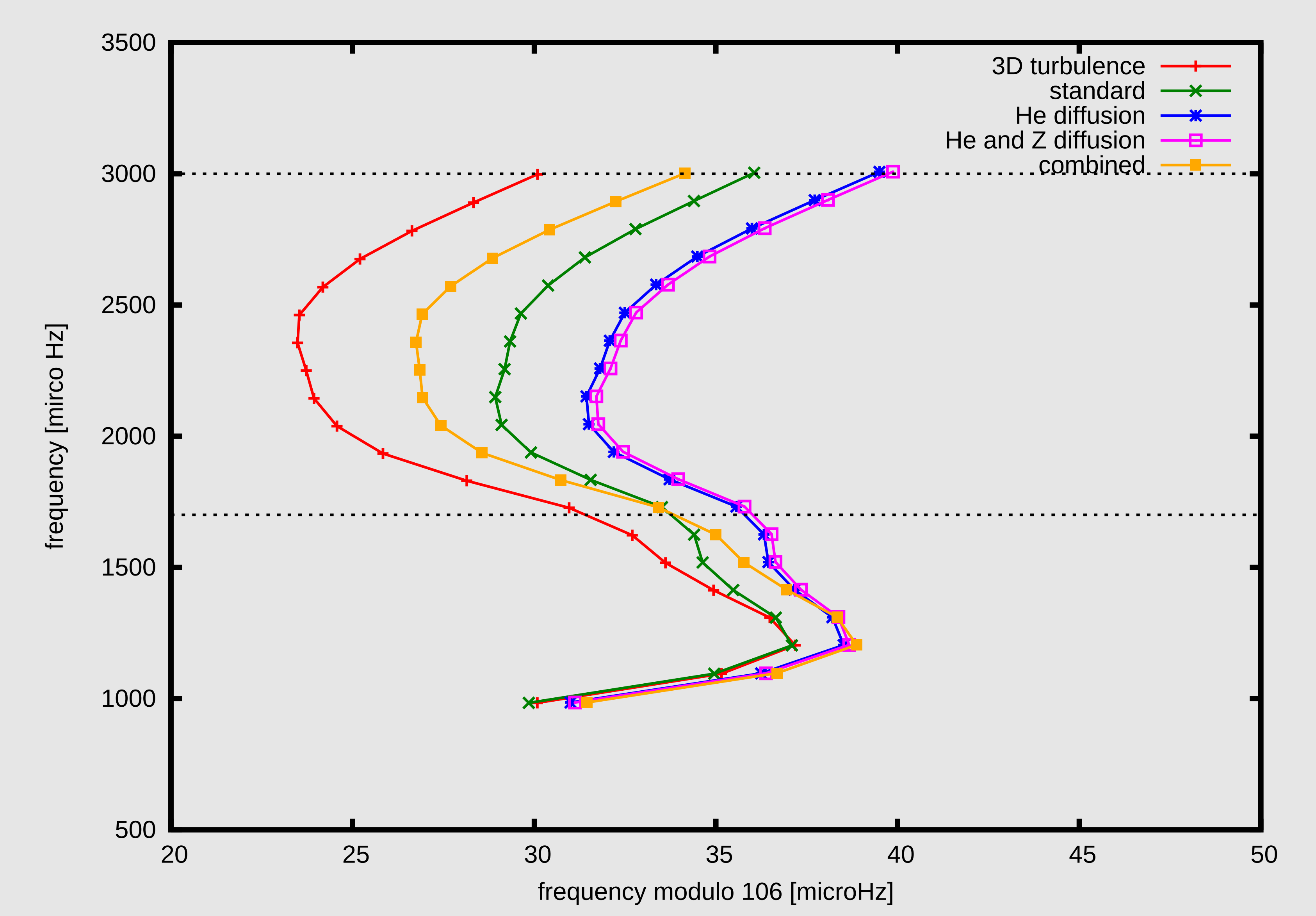
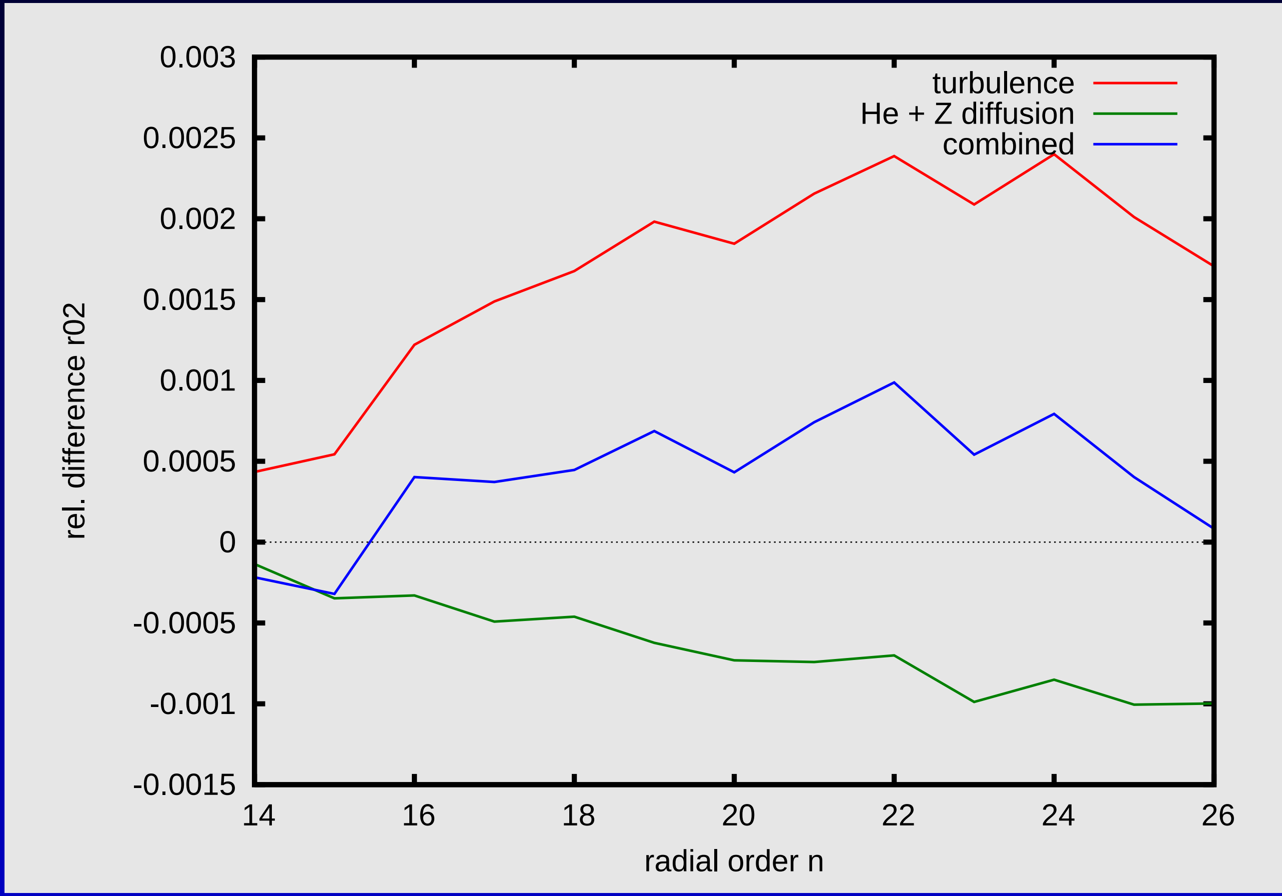


Diagnostics



- $r_{02} = \delta\nu_{00}/\Delta\nu_{01}$ (Roxburgh & Vorontsov (2003))
- affected by less than 0.3%

Conclusions

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- effects may be disentangled
- assumptions: precise L,R and age (age from independent diagnostic)