



**“Tidal evolution of close-in planets
using a Maxwell visco-elastic rheology”**

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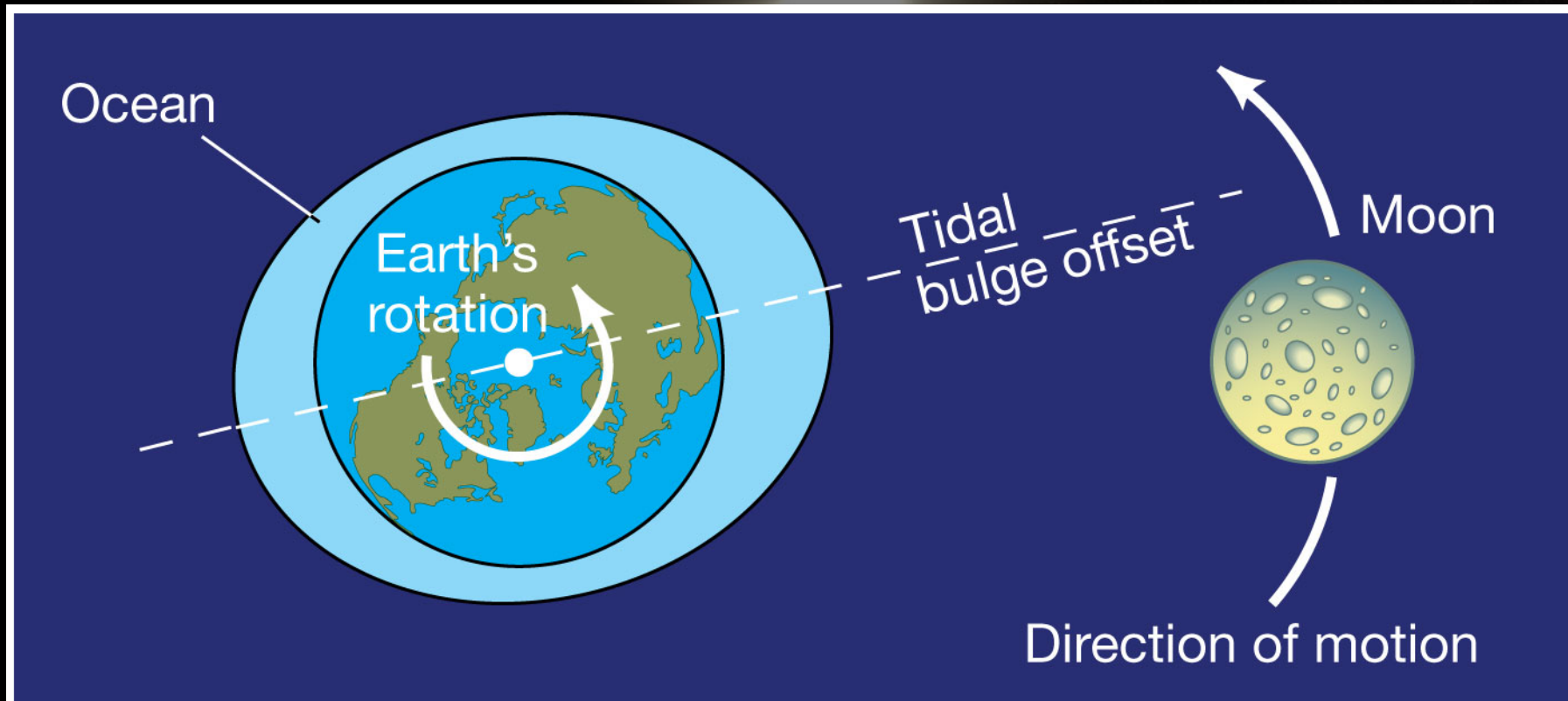
(Univ. Aveiro / IMCCE - Obs. Paris)

G. Boué, J. Laskar, A. Rodríguez

Porto, Portugal

15th - 19th September 2014

Tidal effects



Global Picture:

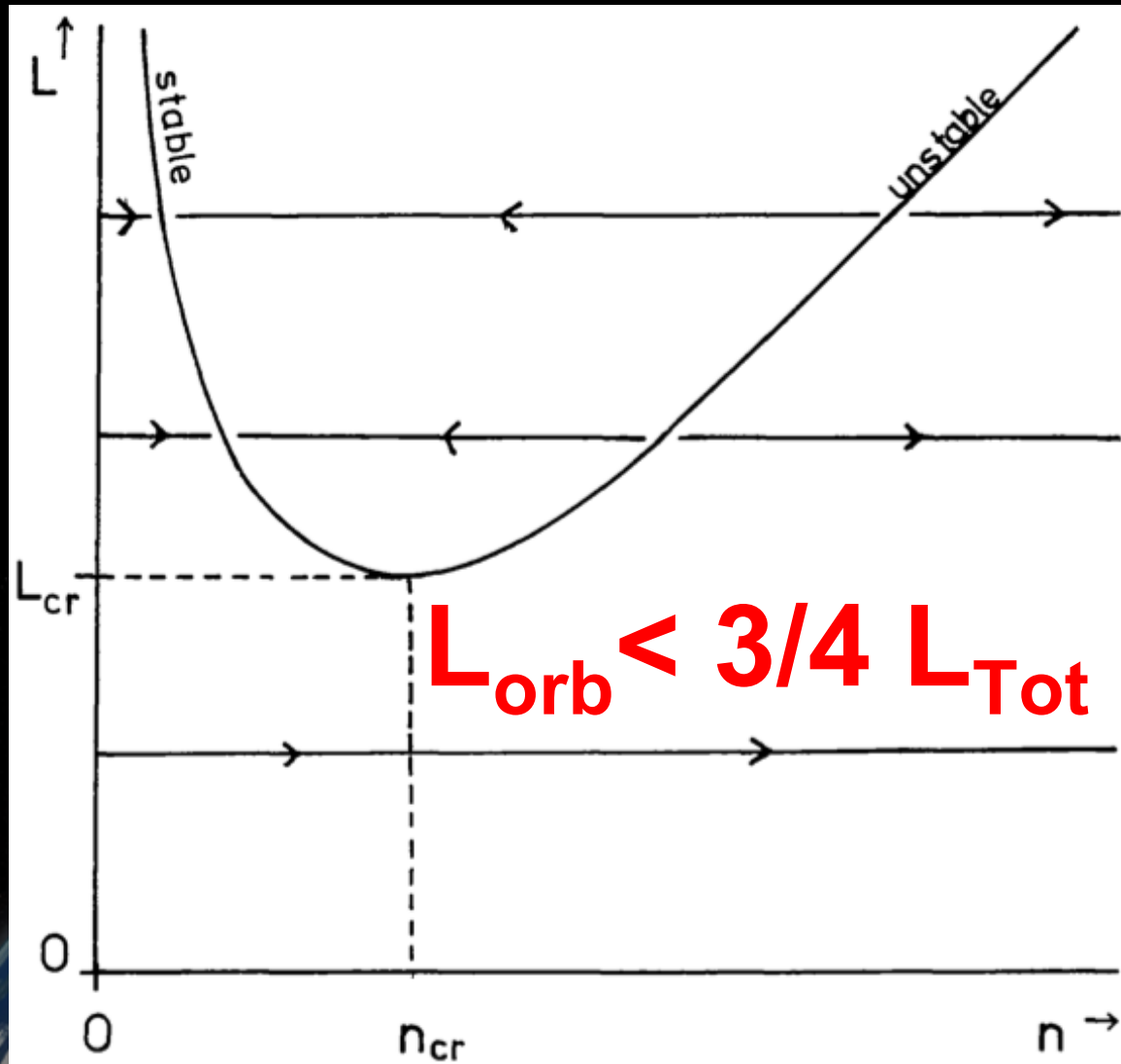
Poincaré (1898)

Landau & Lifshitz (1960)

Hut (1980)

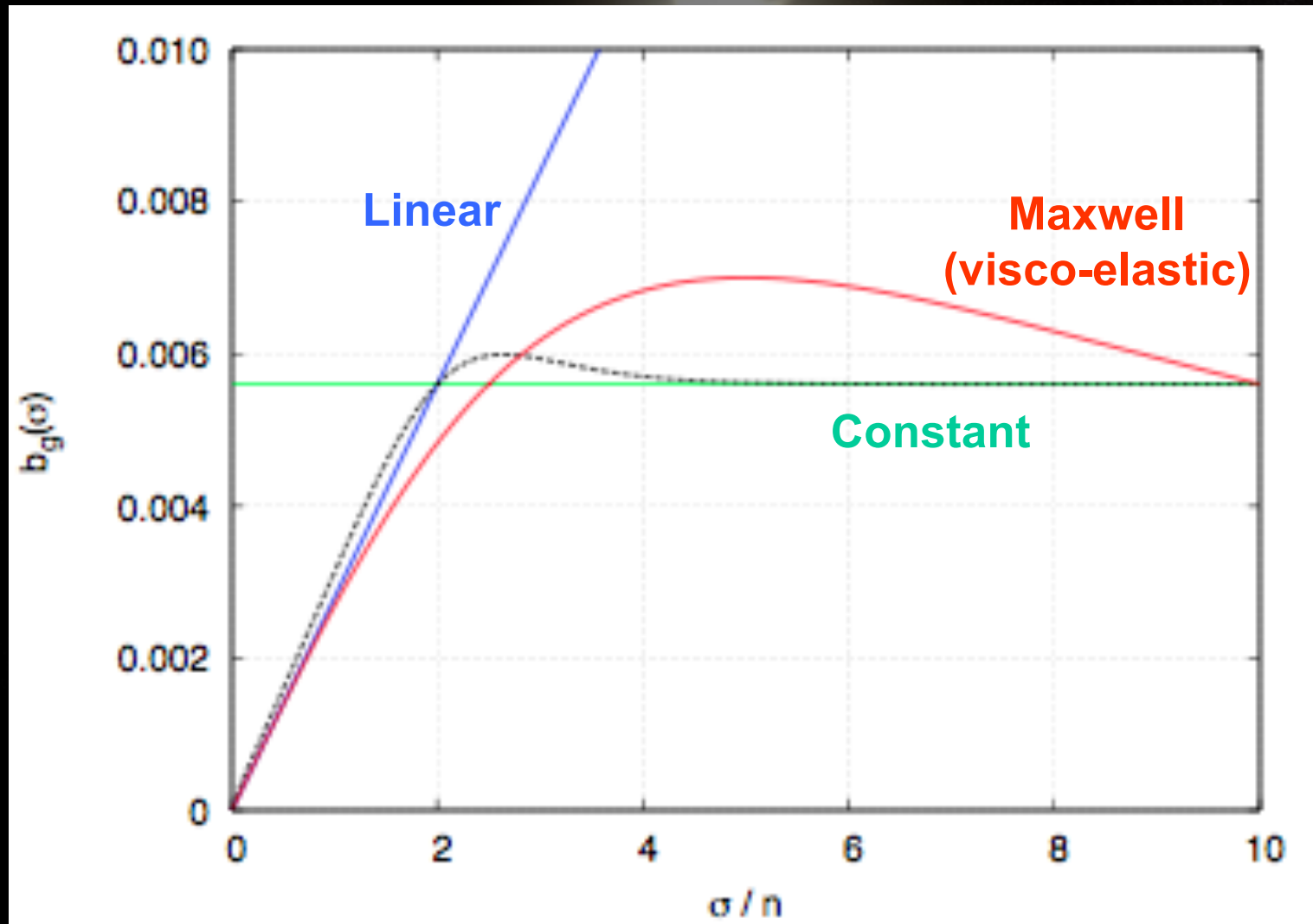
$P_{\text{orb}} = P_{\text{rot}}$
 perpendicular
 axis ($\varepsilon=0$)

circular
 orbits ($e=0$)

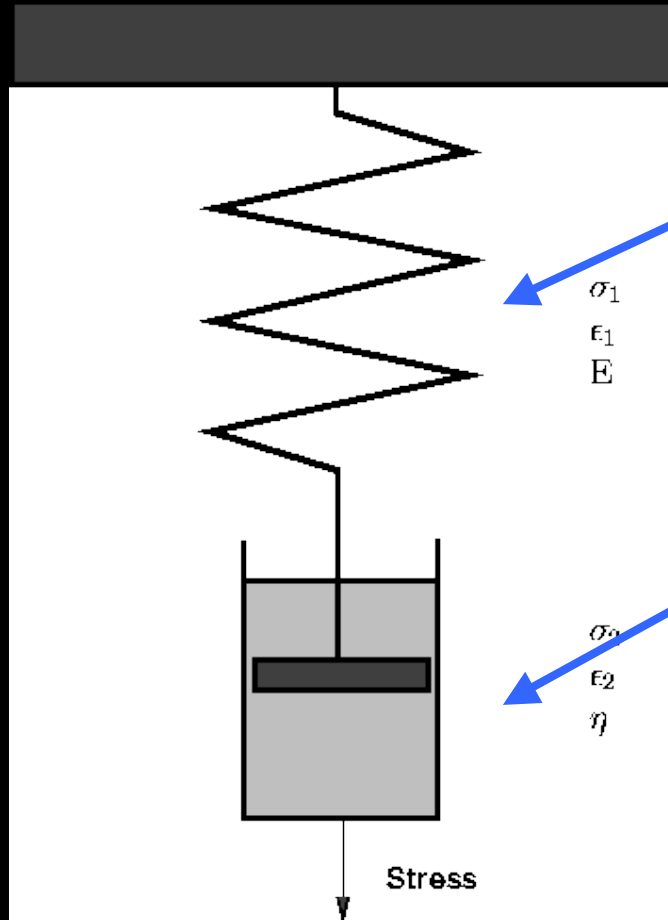


$$L_{\text{cr}} = 4 \left\{ \frac{1}{27} G \frac{M^3 m^3}{M+m} (I_1 + I_2) \right\}^{1/4}$$

Tidal models



Maxwell model (1867)



elastic

$$\sigma = E \epsilon$$

strain

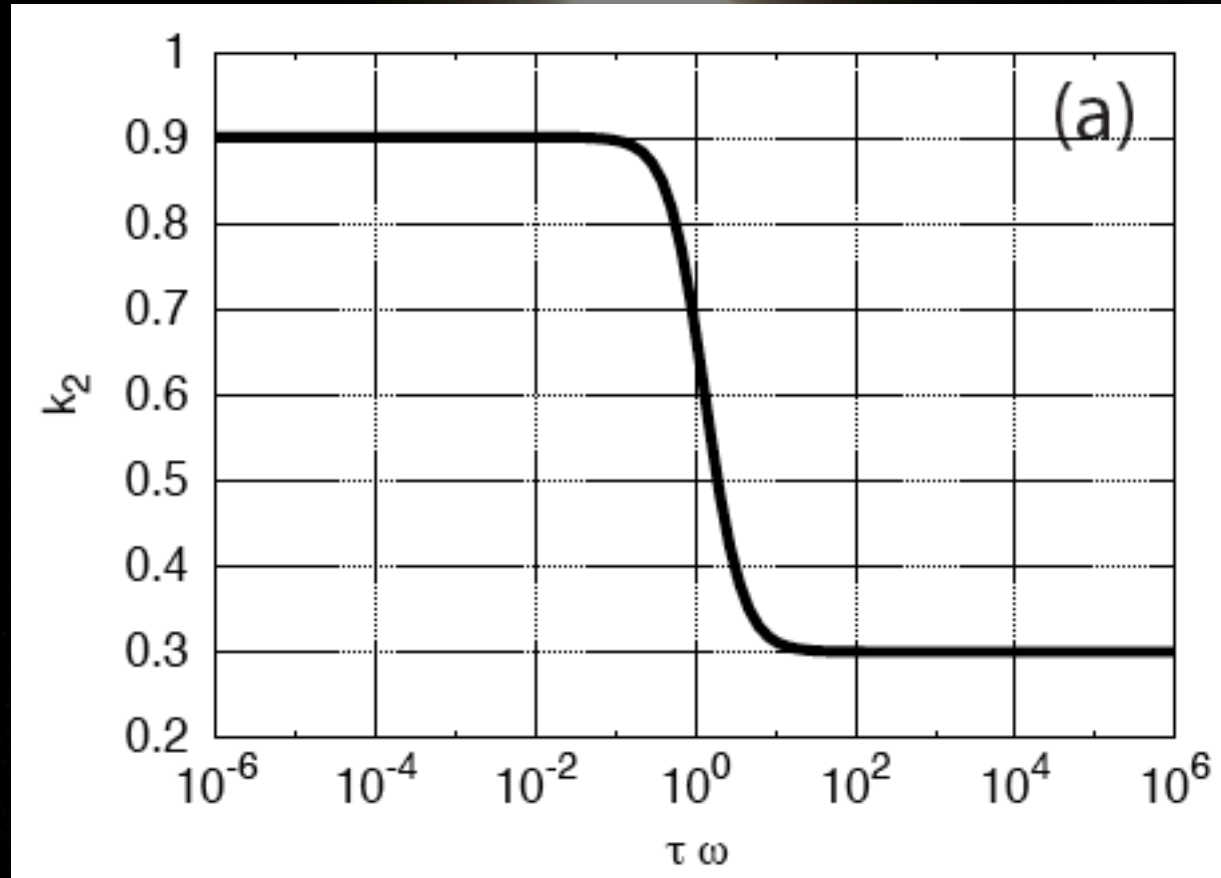
viscous

$$\sigma = \eta d\epsilon/dt$$

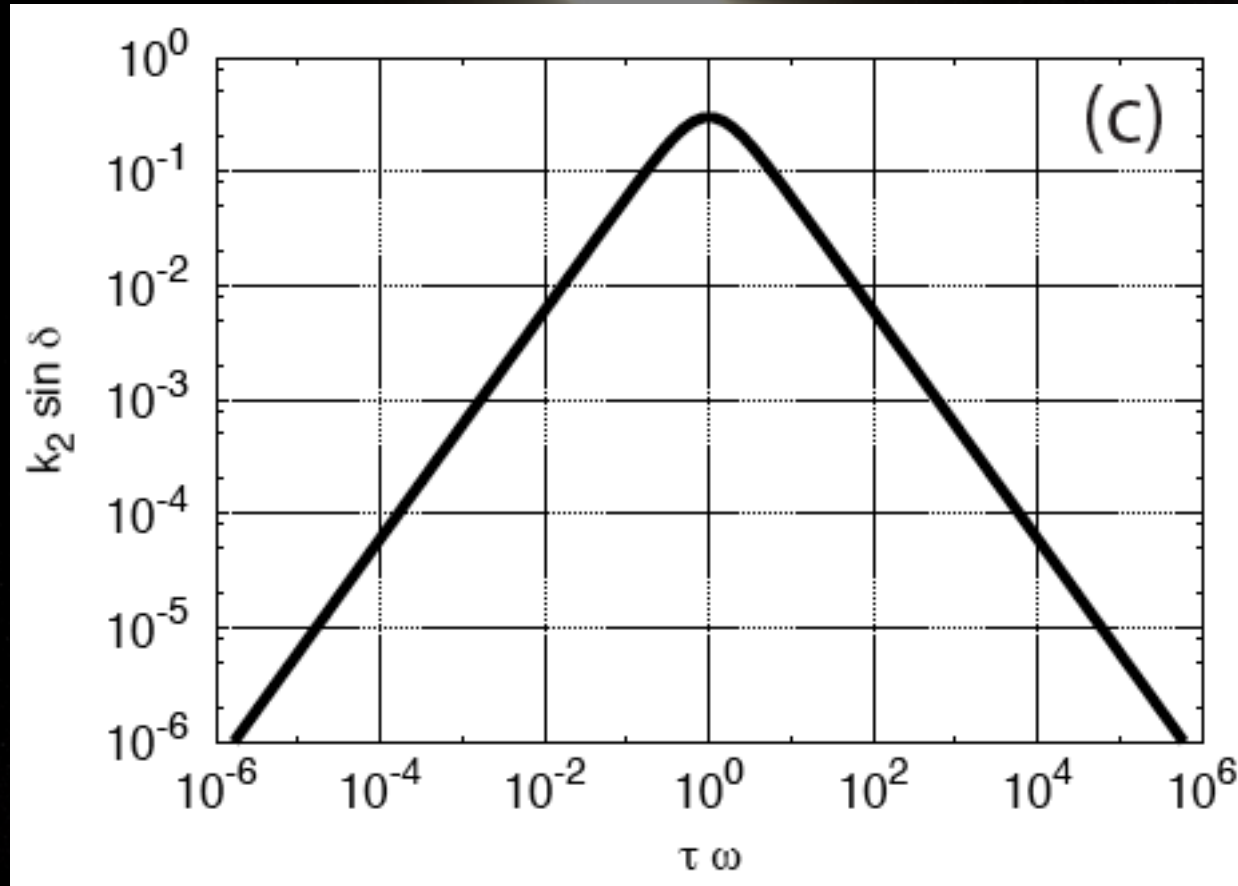
$$\frac{d\epsilon_{\text{Total}}}{dt} = \frac{d\epsilon_D}{dt} + \frac{d\epsilon_S}{dt} = \frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt}$$

$$\frac{1}{E} \frac{d\sigma}{dt} + \frac{\sigma}{\eta} = \frac{d\epsilon}{dt}$$

Deformation



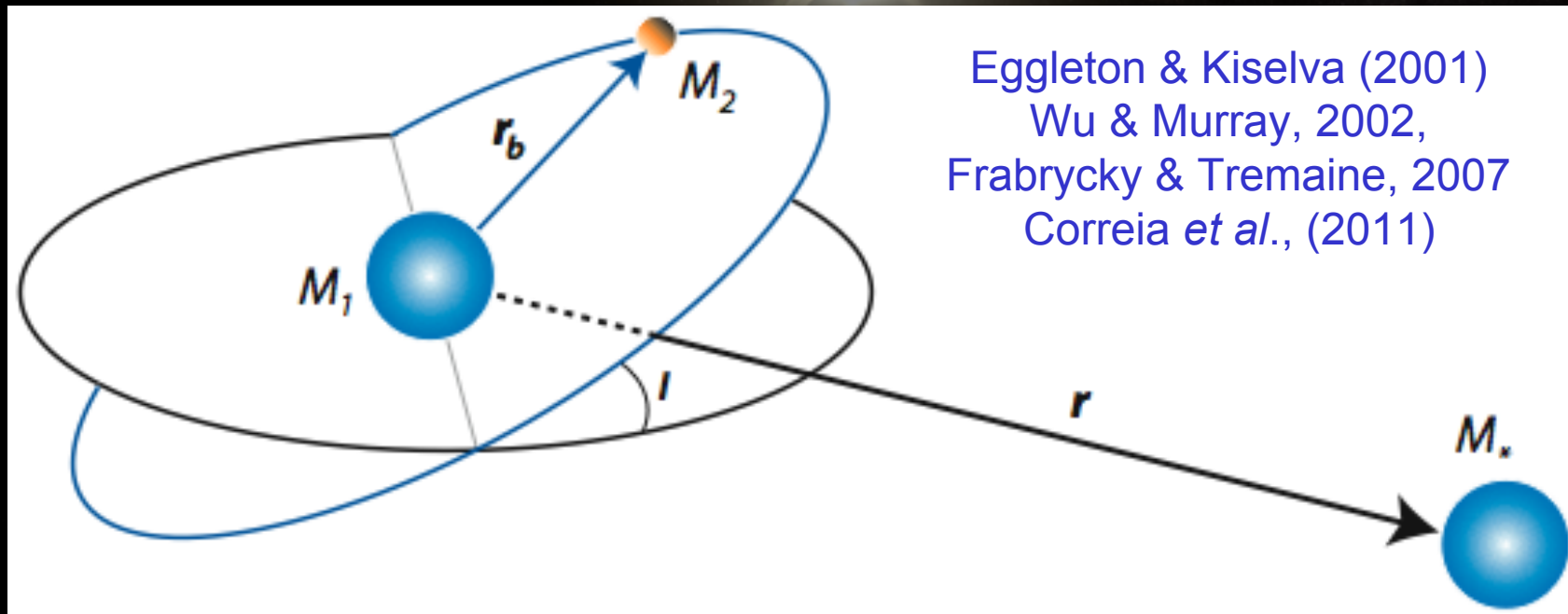
Dissipation



HD 80606

Naef et al, 2001

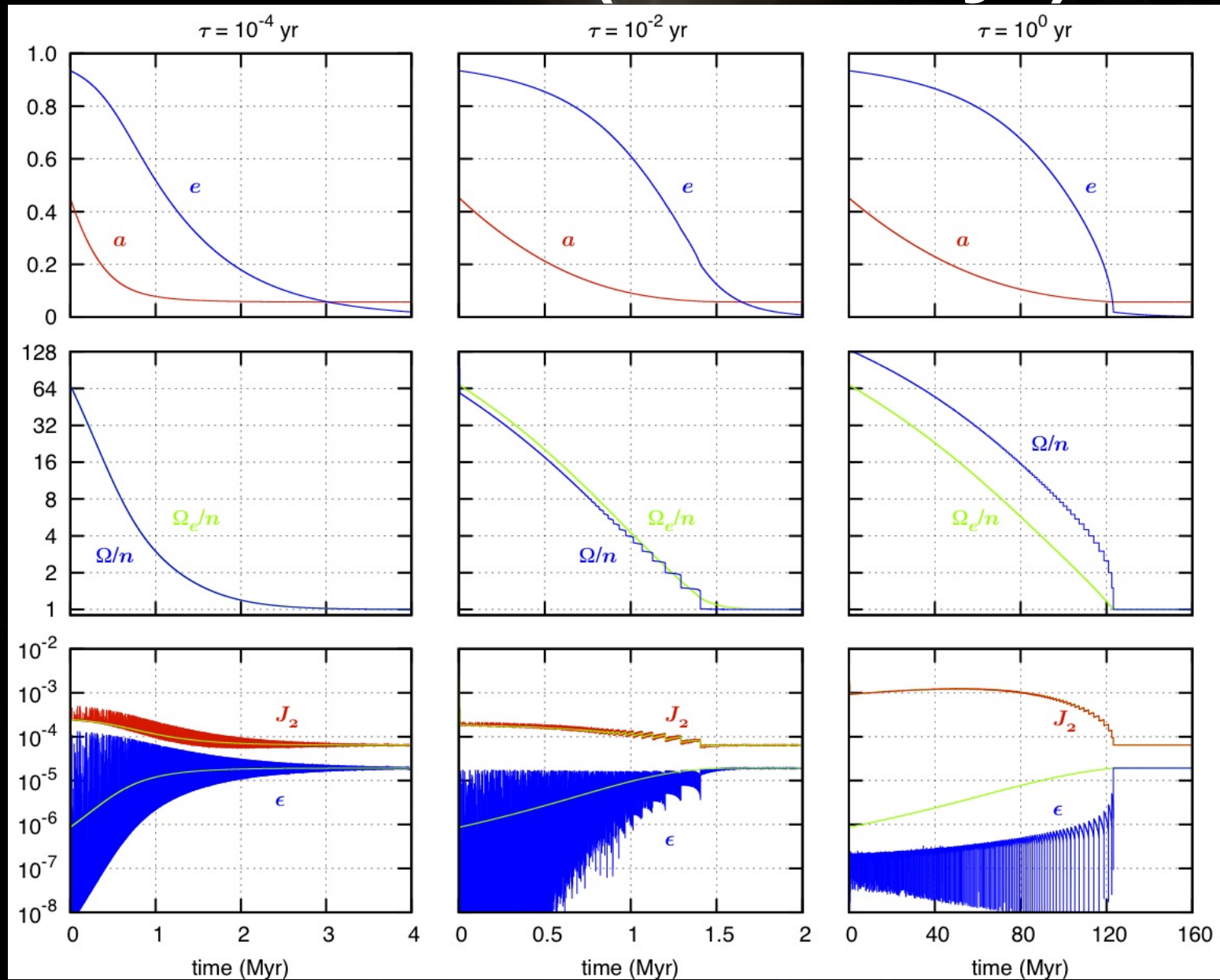
($a_p = 0.45$ AU, $e_p = 0.92$)



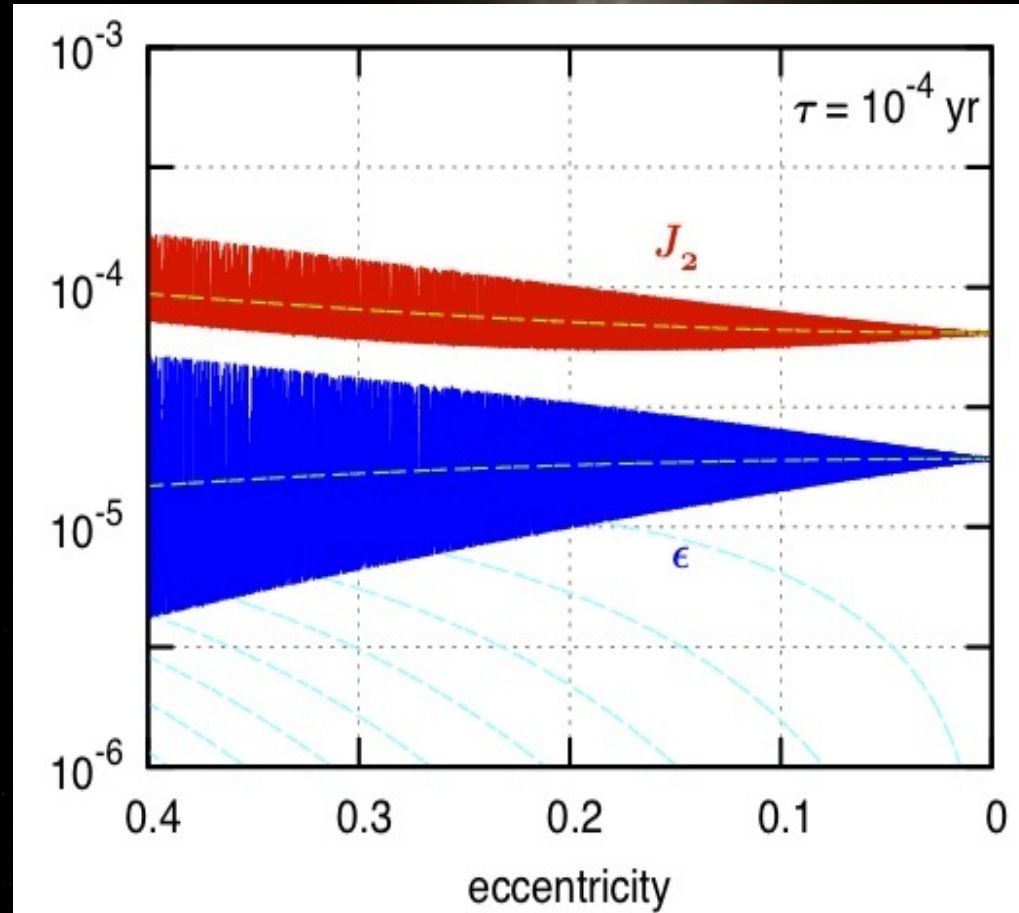
Kozai effect:
($I > 39^\circ$)

$$\sqrt{1 - e_1^2} \cos I = h_1 = Cte$$

HD 80606 ($n \sim 10^{-2}$ yr)



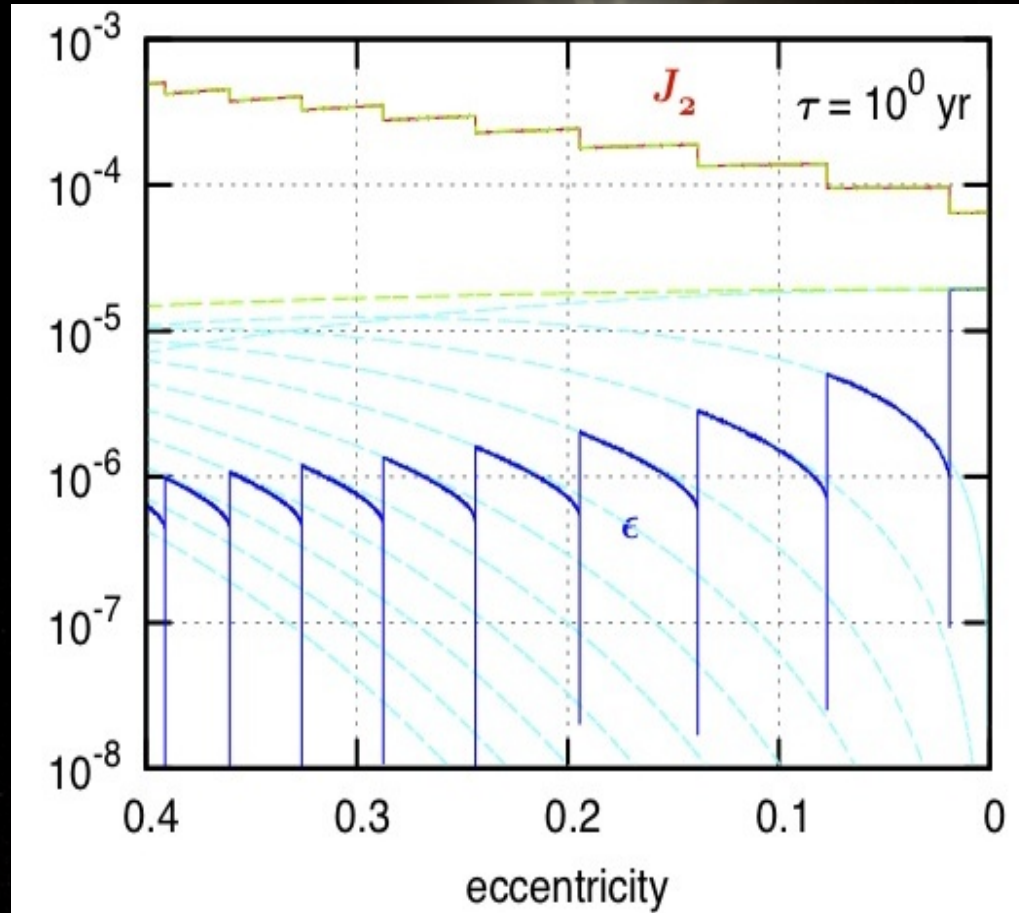
HD 80606 - deformation ($\tau \ll 1/n$)



$$\langle \epsilon \rangle_M = \frac{k_f m_0}{4 m} \left(\frac{R}{a} \right)^3 (1 - e^2)^{-3/2}$$

$$\langle \epsilon_p \rangle_M = \beta_{2p} = \frac{k_f m_0}{4 m} \left(\frac{R}{a} \right)^3 X_{2p}^{-3,2}(e)$$

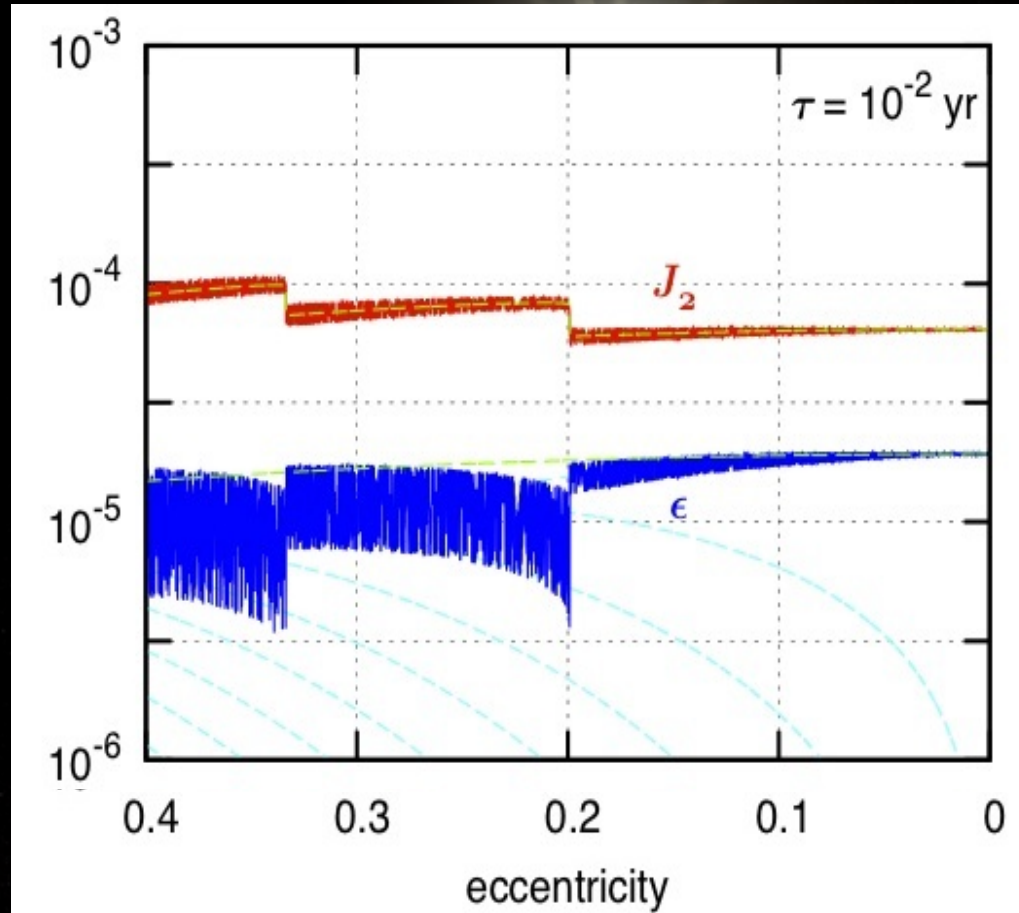
HD 80606 - deformation ($\tau \gg 1/n$)



$$\langle \epsilon \rangle_M = \frac{k_f m_0}{4 m} \left(\frac{R}{a} \right)^3 (1 - e^2)^{-3/2}$$

$$\langle \epsilon_p \rangle_M = \beta_{2p} = \frac{k_f m_0}{4 m} \left(\frac{R}{a} \right)^3 X_{2p}^{-3,2}(e)$$

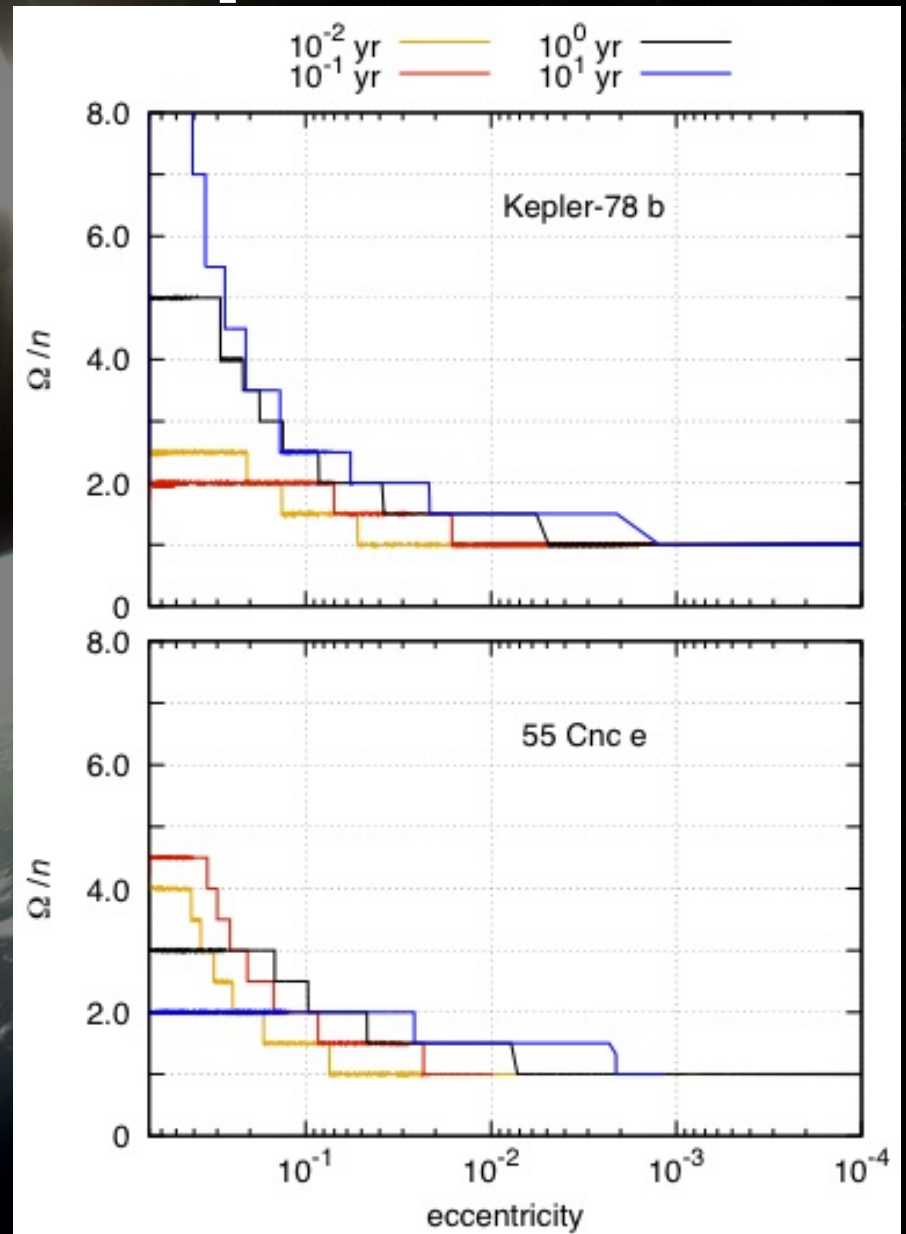
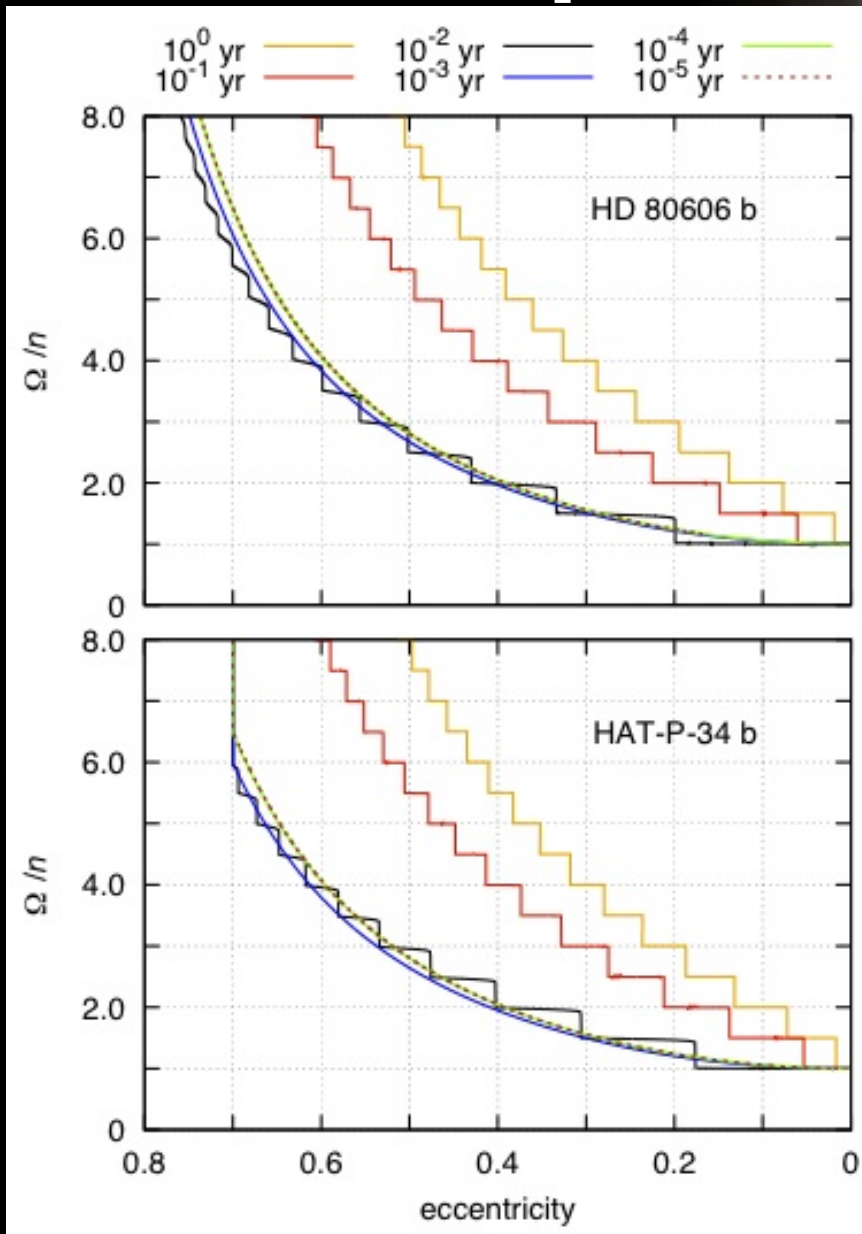
HD 80606 - deformation ($\tau \sim 1/n$)



$$\langle \epsilon \rangle_M = \frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{a} \right)^3 (1 - e^2)^{-3/2}$$

$$\langle \epsilon_p \rangle_M = \beta_{2p} = \frac{k_f}{4} \frac{m_0}{m} \left(\frac{R}{a} \right)^3 X_{2p}^{-3,2}(e)$$

Hot-Jupiters / super-Earths



Conclusions

- **We replaced a Fourier series by a rheological law of deformation of the potential. This allow us to simultaneously take into account deformation and dissipation.**
- **We no longer need to truncate the series for high eccentricities and we avoid a large number of terms. The deformation is also valid for all tidal regimes.**
- **Spin-orbit resonances arise naturally when the deformation time-scale is longer than the orbital period. The stability increases with the deformation time-scale.**
- **For rocky planets, the spin-orbit resonances are possible for very low values of the eccentricity, so we expect that some of these planets are not synchronous with the star.**